EPS 6th Grade Mathematics Instructional Sequence 2010-11

Unit	Start date:	6th Grade Core Content Standard
Bits & Pieces II (CMP2)	Sept. 10	**6.1** *Multiplication and division of fractions and decimals.* Fluently and accurately multiply and divide non-negative fractions.
Covering and Surrounding	Oct. 20	**6.4** *Two-dimensional figures*: area and perimeter (circumference) of various 2-Dshapes, some of which are composite figures.
Integers / Order of Operations	Nov.19	**6.2 / 6.5** *Mathematical expressions*: intro to integers, evaluate algebraic expressions
Bits and Pieces III (CMP2)	Dec. 7	**6.1** *Multiplication and division of fractions and decimals*: Fluently and accurately computing, and problem solving with fractions, decimals, and percents.
Variables and Patterns	Feb.14	**6.2** *Mathematical expressions and equations*: Use of variables along with tables, words, numbers, graphs, and equations to describe linear relationships.
Comparing and Scaling (CMP2)	Mar. 22	**6.3** *Ratios, rates, and percents*: Writing and using ratios to solve problems
Filling and Wrapping	April 27	**6.4** *Two-dimensional figures*: surface area and volume of various 3-D figures.
How Likely Is It?	May 27	**6.3** *Ratios, rates, percents*: determine experimental and theoretical probabilities (ratio form)

Grade 6 Adopted Washington State Mathematics Standards

April 28, 2008

6.1. Core Content:
Multiplication and division of fractions and decimals **(Numbers, Operations, Algebra)**

Students have done extensive work with fractions and decimals in previous grades and are now prepared to learn how to multiply and divide fractions and decimals with understanding. They can solve a wide variety of problems that involve the numbers they see every day—whole numbers, fractions, and decimals. By using approximations of fractions and decimals, students estimate computations and verify that their answers make sense.

6.2. Core Content:
Mathematical expressions and equations **(Operations, Geometry/Measurement, Algebra)**

Students continue to develop their understanding of how letters are used to represent numbers in mathematics—an important foundation for algebraic thinking. Students use tables, words, numbers, graphs, and equations to describe simple linear relationships. They write and evaluate expressions and write and solve equations. By developing these algebraic skills at the middle school level, students will be able to make a smooth transition to high school mathematics.

6.3. Core Content:
Ratios, rates, and percents **(Numbers, Operations, Geometry/Measurement, Data/Statistics/Probability)**

Students extend their knowledge of fractions to develop an understanding of what a ratio is and how it relates to a rate and a percent. Fractions, ratios, rates, and percents appear daily in the media and in everyday calculations like determining the sale price at a retail store or figuring out gas mileage. Students solve a variety of problems related to such situations. A solid understanding of ratios and rates is important for work involving proportional relationships in grade seven.

6.4. Core Content:
Two-dimensional figures **(Geometry/Measurement, Algebra)**

Students extend what they know about area and perimeter to more complex two-dimensional figures, including circles. They find the surface area and volume of simple three-dimensional figures. As they learn about these important concepts, students can solve problems involving more complex figures than in earlier grades and use geometry to deal with a wider range of situations. These fundamental skills of geometry and measurement are increasingly called for in the workplace and they lead to a more formal study of geometry in high school.

6.5. Additional Key Content **(Numbers, Operations)**

Students extend their mental math skills now that they have learned all of the operations—addition, subtraction, multiplication, and division—with whole numbers, fractions, and decimals. Students continue to expand their understanding of our number system as they are introduced to negative numbers for describing positions or quantities below zero. These numbers are a critical foundation for algebra, and students will learn how to add, subtract, multiply, and divide positive and negative numbers in seventh grade as further preparation for algebraic study.

6.6. Core Processes:
Reasoning, problem solving, and communication

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade six, students solve problems that involve fractions and decimals as well as rates and ratios in preparation for studying proportional relationships and algebraic reasoning in grade seven.

Mathematics content based on
Adopted Washington State
K-8 Mathematics Standards,
April 28, 2008,
Office of the Superintendent of Public Instruction

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Grade Six Performance Expectations

Multiplication and Division of Fractions and Decimals (numbers, operations, algebra)

6.1.A **Compare and order non-negative fractions, decimals,** *and integers* **using the number line, lists, and the symbols <, >, or =.** (4.2.E)

6.1.B *Represent multiplication and division of non-negative fractions and decimals using area models and the number line, and connect each representation to the related equation.* (7.1.B)

6.1.C **Estimate products and quotients of fractions and decimals.** ** (5.1.D)

6.1.D *Fluently and* **accurately multiply and divide non-negative fractions** *and explain the inverse relationship between multiplication and division with fractions.* (7.1.C)

6.1.E **Multiply and divide whole numbers and decimals by 1000, 100, 10, 1, 0.1, 0.01, and 0.001.** (5.1.B)

6.1.F *Fluently and* **accurately multiply and divide non-negative decimals.** (4.1.F) (5.1.C) (7.1.C)

6.1.G **Describe the effect of multiplying or dividing a number by one, by zero, by a number between zero and one, and by a number greater than one.** **

6.1.H **Solve single- and multi-step word problems involving operations with fractions and decimals and verify the solutions.** (5.2.H) (7.1.G)

> The performance expectation identified in the parentheses represents a connection to a previous or future grade level performance expectation.
>
> **Bold** and *italicized* formatting based on the most current version of the MSP Mathematics Item Specifications. Expectations for the state assessment are in **bold text**. Expectations for local instruction and assessment appear in *italicized* text.
> ** This performance expectation may be included in items assessing process performance expectaions.

Mathematical Expressions and Equations (operations, geometry/measurement, algebra)

6.2.A **Write a mathematical expression or equation with variables to represent information in a table or given situation.** (5.4.B) (5.4.C) (7.1.F)

6.2.B **Draw a first-quadrant graph in the coordinate plane to represent information in a table or given situation.** (5.4.D)

6.2.C **Evaluate mathematical expressions when the value for each variable is given.** (5.4.C)

6.2.D **Apply the commutative, associative, and distributive properties, and use the order of operations to evaluate mathematical expressions.** (8.4.C)

6.2.E **Solve one-step equations and verify solutions.** (7.1.E)

6.2.F **Solve word problems using mathematical expressions and equations and verify solutions.**

Ratios, rates, and percents (numbers, operations, geometry/measurement, data/statistics/probability)

6.3.A **Identify** *and write* **ratios as comparisons of part-to-part and part-to-whole relationships.**

6.3.B **Write ratios to represent a variety of rates.**

6.3.C **Represent percents visually and numerically, and convert between the fractional, decimal, and percent representations of a number.**

6.3.D **Solve single- and multi-step word problems involving ratios, rates, and percents, and verify the solutions.**

6.3.E *Identify the ratio of the circumference to the diameter of a circle as the constant* π*, and recognize 22/7 and 3.14 as common approximations of pi.*

6.3.F **Determine the experimental probability of a simple event using data collected in an experiment.** (4.4.H)

6.3.G **Determine the theoretical probability of an event and its complement and represent the probability as a fraction or decimal from 0 to 1 or as a percent from 0 to 100.** (4.4.G) (7.4.A) (7.4.B)

Two-Dimensional Figures (geometry/measurement, algebra)

6.4.A **Determine the circumference and area of circles.** (7.3.A)

6.4.B **Determine the perimeter and area of a composite figure that can be divided into triangles, rectangles, and parts of circles.** (5.3.F)

6.4.C **Solve single- and multi-step word problems involving the relationships among radius, diameter, circumference, and area of circles, and verify the solutions.** (7.2.B) (7.3.D)

6.4.D **Recognize** *and draw* **two-dimensional representations of three-dimensional figures.**

6.4.E **Determine the surface area and volume of rectangular prisms using appropriate formulas** *and explain why the formulas work.* (7.3.A)

6.4.F **Determine the surface area of a pyramid.** (7.3.B)

6.4.G **Describe and sort polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.**

Additional Key Content (numbers, operations)

6.5.A *Use strategies for mental computations with non-negative whole numbers, fractions, and decimals.* (5.1.E) (7.2.A)

6.5.B **Locate positive and negative integers on the number line and use integers to represent quantities in various contexts.**

6.5.C **Compare and order positive and negative integers using the number line, lists, and the symbols <, >, or =.** (4.2.E) (7.1.A)

Reasoning, Problem Solving, and Communication

6.6.A *Analyze a problem situation to determine the question(s) to be answered.* (5.6.A) (7.6.A)

6.6.B **Identify relevant, missing, and extraneous information related to the solution to a problem.** (5.6.B) (5.6.C) (7.6.B)

6.6.C *Analyze and compare mathematical strategies for solving problems, and* **select and use one or more strategies to solve a problem.** (5.6.D) (5.6 E) (7.6.C)

6.6.D **Represent a problem situation,** *describe the process used to solve the problem, and* **verify the reasonableness of the solution.** (5.6.F) (5.6.G) (5.6.H) (7.6.D)

6.6.E *Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.* (5.6.I) (7.6.E)

6.6.F *Apply a previously used problem-solving strategy in a new context.* (5.6.D) (7.6.F)

6.6.G **Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.** (5.6.I) (7.6.G)

6.6.H *Make and test conjectures based on data (or information) collected from explorations and experiments.* (5.6.J)(7.6.H)

Bits & Pieces II (CMP2)

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations
				6.1.A Compare and order non-negative fractions, decimals, and integers using the number line, lists, and the symbols <, >, or =. *6.1.B Represent multiplication and division of non-negative fractions and decimals using area models and the number line, and connect each representation to the related equation.* **6.1.C Estimate products and quotients of fractions and decimals.** **6.1. Fluently and accurately multiply and divide non-negative fractions and explain the inverse relationship between multiplication and division with fractions.** **6.1.H Solve single- and multi-step word problems involving operations with fractions and decimals and verify the solutions.** *6.5.A Use strategies for mental computations with non-negative whole numbers, fractions, and decimals.* Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
(CMP2) Prob. 2.1 Writing Addition and Subtraction Sentences (Land Problem)	2			
(CMP2) Prob. 2.2 Using addition and subtraction pg. 20	1			
(CMP2) Prob. 2.4 Designing Algorithms pg. 23	1			
(CMP2) Skill Sheets Estimating Inv. 1 & Adding/ Subtracting Inv. 2 & Inv. 2 Math Reflection pg. 31	2			
Check –up # 2	1			
(CMP2) Prob. 3.1 A model for multiplication	2			
(CMP2) Prob. 3.2 Another model for multiplication	1			
(CMP2) Prob. 3.3 Modeling more multiplication Situations	1			
(CMP2) Prob. 3.4 Multiplication with Mixed Numbers	1			
(CMP2) Prob. 3.5 Writing a Multiplication Algorithm	2			
More Practice with Multiplication of Fractions & *(CMP2)* Inv. 3 Math Reflections	2			
(CMP2)**Check- up #3, Partner Quiz or combo of the two**	1			
(CMP2) Prob. 4.1 Dividing Fractions/Preparing Food	2			
(CMP2) Prob. 4.2 Dividing a Fraction by a Whole Number	1			
(CMP2) Prob. 4.3 Dividing a Fraction by a Fraction	1			
(CMP2) Prob. 4.4 Writing a Division Algorithm	2			
Finish Prob. 4.4 *(CMP2)* from the day before & More practice with division of fractions	2			
(CMP2) Inv. 4 Math Reflections	1			
Bits and Pieces II Unit Assessment	1			
Total Instructional Days for Bits and Pieces II :	**27 days**			

All page numbers given match the student texts

Contents in Bits and Pieces II CMP 2

- There isn't any supplemental material for this unit

Covering and Surrounding

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations /Additional Targets
				6.3.E Identify the ratio of the circumference to the diameter of a circle as the constant π, and recognize 22/7 and 3.14 as common approximations of pi.
(CMP2) **ACE questions pgs. 11- 13 select questions from #2-7, 9-23 (review finding area & perimeter of rectangles first then assign ACE problems with emphasis on problems 9-12, 22-23)**	1	CMP2 Disk(6)		
(CMP2) Prob. 3.1 Finding Area and Perimeter of Triangles p. 38	1	CMP2 Disk(6)		**6.4.A Determine the circumference and area of circles.**
(CMP2) Prob. 3.2 Identifying Base and Height p. 41	1			
CMP2) Prob. 4.1 Finding Measures of Parallelograms p. 54	1			**6.4.B Determine the perimeter and area of a composite figure that can be divided into triangles, rectangles, and parts of circles.**
Practice with MSP formula sheet for parallelograms and triangles and Quick Quiz (in binder)	2			
Prob. 7.1 Going Around in Circles pg. 69	2		optional	**6.4.C Solve single- and multi-step word problems involving the relationships among radius, diameter, circumference, and area of circles, and verify the solutions.**
Prob. 7.2 Surround a Circle pg. 71	2		must do	
Prob. 7.3 & 7.4 Covering Circles pg. 72-73	2		must do	
Area and Perimeter of Composite Figures activity (in binder)	1			
Inv. 7 Mathematical Reflections pg. 81 & Practice with circumference, area, and relationships with Circles (6.4.C) (in binder)	3			Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Covering & Surrounding Unit Review	1			
Covering & Surrounding Unit Assessment	1			
Total Instructional Days for Covering and Surrounding: 18 days				

All page numbers given match the student texts.

Contents in Covering and Surrounding

- ACE questions pp.11-13
- ACE questions pp.11-13 Answers
- CMP2 Covering & Surrounding: Investigation 3.1 SE
- CMP2 Covering & Surrounding: Investigation 3.1 TE
- CMP2 Covering & Surrounding: Investigation 3.2 SE
- CMP2 Covering & Surrounding: Investigation 3.2 TE
- CMP2 Covering & Surrounding: Investigation 4.1 SE
- CMP2 Covering & Surrounding: Investigation 4.1 TE
- Covering and Surrounding Quiz
- Area and Perimeter of Composite Figures worksheet
- Area and Circumference of Circles Extension Problems
- MSP formula sheet

For Exercises 2–5, experiment with tiles or square grid paper. Sketch each answer on grid paper.

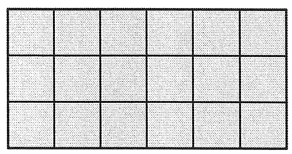
2. Draw two different shapes with an area of 16 square units. What is the perimeter of each shape?

3. Draw two different shapes with a perimeter of 16 units. What is the area of each shape?

4. Draw two different shapes with an area of 6 square units and a perimeter of 12 units.

5. Draw two different shapes with an area of 15 square units and a perimeter of 16 units.

6. Use this design for parts (a) and (b).

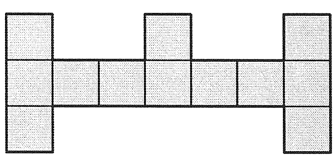


a. If possible, draw a figure with the same area, but with a perimeter of 20 units. If this is not possible, explain why.

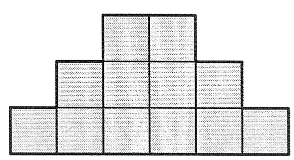
b. If possible, draw a figure with the same area, but with a perimeter of 28 units. If this is not possible, explain why.

7. These designs have an area of 12 square meters. Are the perimeters the same? Explain how you decided.

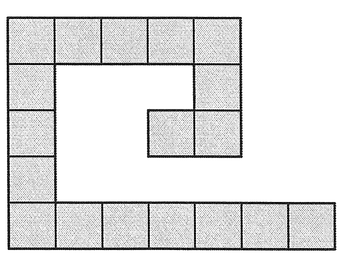
Design C



Design F



8. Copy the design below onto grid paper. Add six squares to make a new design with a perimeter of 30 units. Explain how the perimeter changes as you add tiles to the figure.



For Exercises 9–12, each unit length represents 12 feet. Find the area and perimeter of each floor plan.

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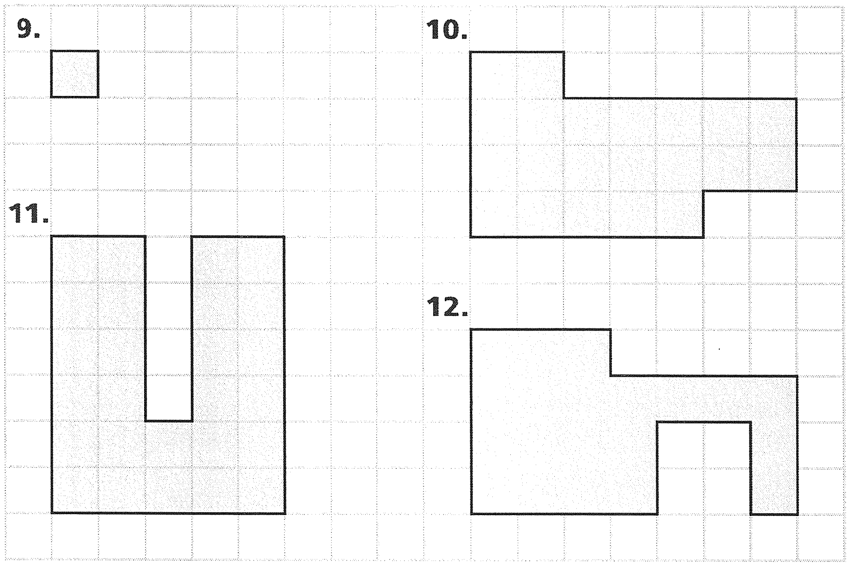
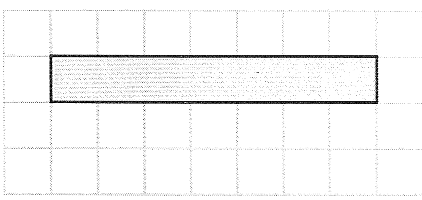
Web Code: ama-5154



For Exercises 13–20, find the area and perimeter of each shaded rectangle.

13.



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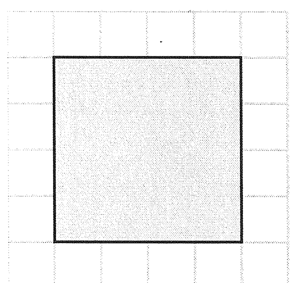

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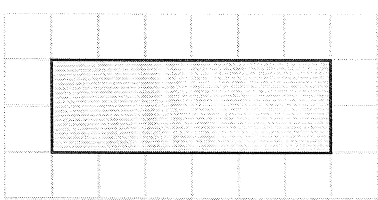

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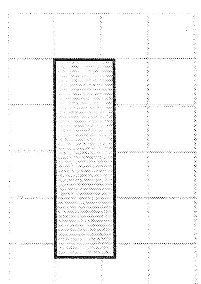

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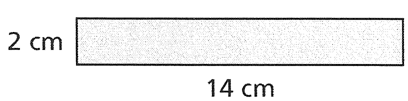

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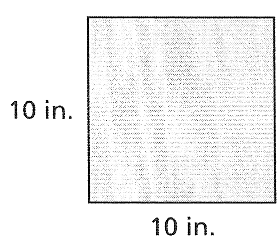

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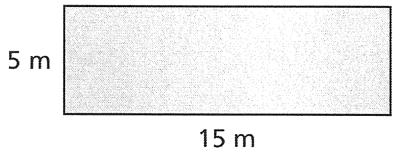

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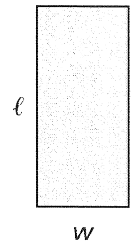


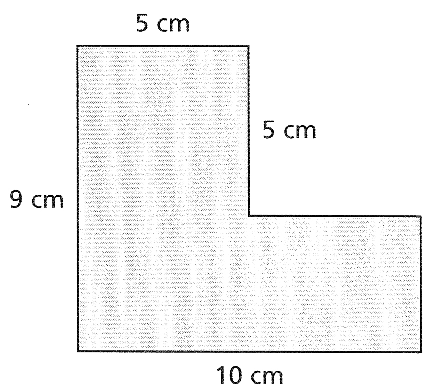
21. Copy and complete the table. Sketch each rectangle and label its dimensions.

Rectangle Area and Perimeter

Rectangle	Length	Width	Area	Perimeter
A	5 in.	6 in.		
B	4 in.	13 in.		
C	$6\frac{1}{2}$ in.	8 in.		

For Exercises 22 and 23, find the area and perimeter of each figure. Figures are not drawn to scale.

22.



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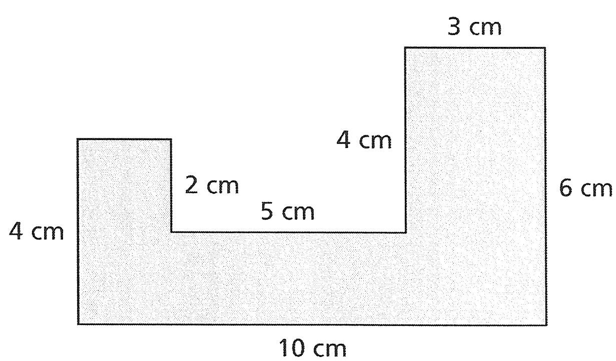


Investigation 1

ACE Assignment Choices

Differentiated Instruction

Problem 1.1
Core 1–5
Other *Application* 6

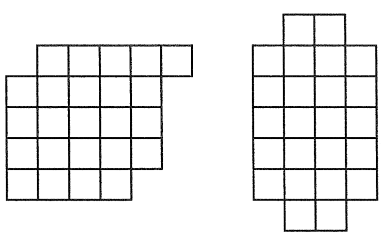
Problem 1.2
Core 7, 9–15, 28
Other *Applications* 8; *Connections* 29, 30, 34, 35; *Extensions* 39, 40; unassigned choices from previous problems

Problem 1.3
Core 16–21, 31
Other *Applications* 22–27; *Connections* 32, 33, 36–38; *Extensions* 41, 42; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 6 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 31, 34: *Prime Time*; 34: *Bits and Pieces I*

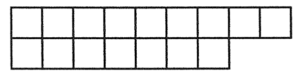
Applications

1. a. Possible answers:

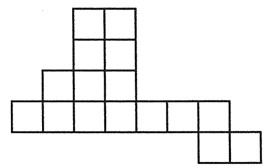


   b. The bumper-car ride has an area of 24 m^2, which is the total number of square meters used to cover the floor plan of the bumper-car ride. The perimeter of 22 m is the total number of rail sections that are needed to surround the bumper-car ride.

2. Answers will vary. Maximum perimeter for whole-number dimensions is 34 units, minimum is 16 units. Possible answers:

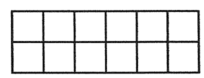


perimeter: 22 units

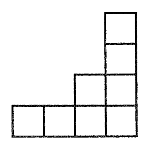


perimeter: 26 units

3. Answers will vary. Maximum area for whole-number dimensions is 16 square units, minimum is 7 square units. Possible answers:

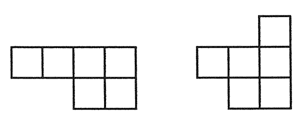


area: 12 square units

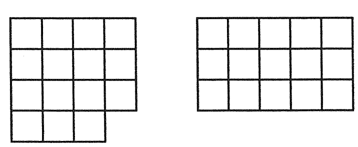


area: 8 square units

4. Answers will vary. Possible answers:

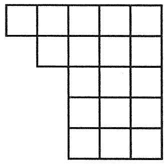


5. Answers will vary. Possible answers:

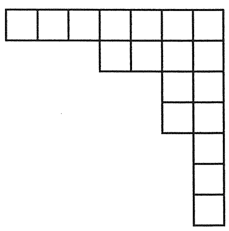


ACE ANSWERS 1

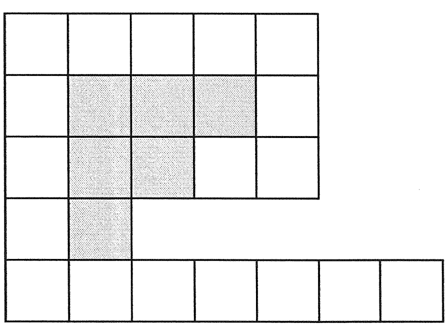
6. a. This is possible. (Note that the area of the original rectangle is 18 square units.) Examples will vary. Possible answer:



b. This is possible. Examples will vary. Possible answer:



7. Possible answers: No, because I counted the number of units around the edge of each figure and found that their perimeters were different. Or, Design J has the maximum perimeter for designs with this area, while Design K has some interior tiles and so must have less than the maximum perimeter.

8. 

Adding these six tiles reduced the perimeter of the figure. Only two of the new tiles have exposed edges, while together they cover ten previously exposed edges in the original figure.

9. $P = 4 \times 12$ ft $= 48$ ft, $A = 12$ ft $\times 12$ ft $= 144$ ft^2

10. $P = 22 \times 12$ ft $= 264$ ft,
$A = 144$ ft$^2 \times 21 = 3{,}024$ ft^2

11. $P = 30 \times 12$ ft $= 360$ ft,
$A = 144$ ft$^2 \times 26 = 3{,}744$ ft^2

12. $P = 26 \times 12$ ft $= 312$ ft,
$A = 144$ ft$^2 \times 20 = 2{,}880$ ft^2

13. $P = 16$ units, $A = 7$ units^2

14. $P = 16$ units, $A = 16$ units^2

15. $P = 16$ units, $A = 12$ units^2

16. A $1\frac{1}{4}$ unit-by-$4\frac{1}{4}$ unit rectangle has $P = 11$ units, $A = 5\frac{5}{16}$ units^2.

17. $A = 28$ cm^2, $P = 32$ cm

18. $A = 100$ in.^{2}, $P = 40$ in.

19. $A = 75$ m^2, $P = 40$ m

20. $A = \ell w$, $P = \ell + \ell + w + w$ or $P = 2\ell + 2w$ or $P = 2(\ell + w)$

21. (Figure 2) Check students' sketches.

22. $A = 65$ cm^2, $P = 38$ cm

23. $A = 36$ cm^2, $P = 36$ cm

24. a. Possible answer: You could draw two imaginary horizontal lines across the room, dividing the floor into three rectangles: one by the door, one by the nook by the window, and a large one taking up the majority of the floor's surface. You would then measure the length and width of each rectangle (in yards) and multiply the two measurements to find the areas (in square yards) of each rectangle. Add the areas together, and multiply the sum by the cost per square yard.

Alternatively, you could get a square that is 1 yd by 1 yd and lay it over the room and to find about how many it would take to cover it. You could take that number and multiply it by the cost for each square yard.

Figure 2

Rectangle	Length (in.)	Width (in.)	Area (square in.)	Perimeter (in.)
A	5	6	30	22
B	4	13	52	34
C	$6\frac{1}{2}$	8	52	29

b. Possible answer: You could lay a ruler all around the base edges of the walls of the room, counting as you go. You would multiply the total by the cost per foot.

Alternatively, you could measure the length of each wall and add the results.

25. **a.** $6 \text{ ft} \times 8\frac{1}{2} \text{ ft} = 51 \text{ ft}^2$

b. 29 ft of molding

c. Two walls have an area of $6 \text{ ft} \times 6 \text{ ft} = 36 \text{ ft}^2$, and two walls have an area of $6 \text{ ft} \times 8\frac{1}{2} \text{ ft} = 51 \text{ ft}^2$. The total surface area would be $36 + 36 + 51 + 51 = 174 \text{ ft}^2$. You would need 4 pints of paint because $174 \text{ ft}^2 \div 50 \text{ ft}^2 = 3.48$, and you round up to 4 so that you will have enough paint.

d. Answers will vary.

26. **a.** $40 \times 120 = 4{,}800$. $4{,}800 \times \$95 = \$456{,}000$

b. $4{,}800 \div 100 = 48$ cars

27. Yes, both rules are correct. The only difference is that Chuck multiplied the lengths and widths by 2 and then added them, and Ruth added the length and width first and then multiplied their sum by 2.

Connections

28. C

29. Possible answers: A tile on the classroom floor is about 1 ft^2. A coffee table is about 1 yd^2.

30. **a.** 1 yd^2 is larger. It is 9 ft^2.

b. They are the same length. 5×12 in. = 60 in.

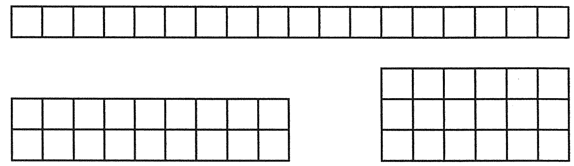
c. 12 m is larger because 120 cm is 1.2 m.

d. 120 ft is larger because 12 yd is 36 ft.

e. They are the same length. 50 cm = 500 mm.

f. One square meter is larger because a meter is longer than a yard.

31. **a.**



b. (Figure 3) **c.** (Figure 4)

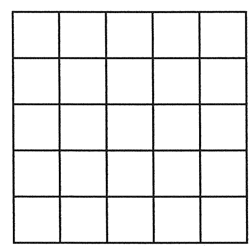
d. The factors of a number and the dimensions of the rectangles that can be made from that number of tiles are the same. For example, the factors of 25 are 1, 5, and 25, so each number can be at least one dimension of a rectangle with 25 square units of area.

32. **a.** $31\frac{45}{100}$ or $31\frac{9}{20}$ **b.** 50

c. $23\frac{29}{32}$ **d.** $1\frac{11}{24}$

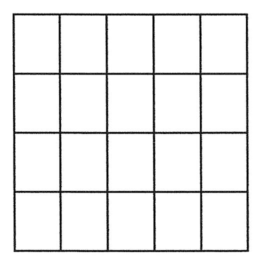
33. **a.** 8 **b.** 16 **c.** 6

34. **a.** Possible answer:



Each brownie is 2 in. by 2 in.

b. Possible answer:



Each brownie is $2\frac{1}{2}$ in. by 2 in.

Figure 3

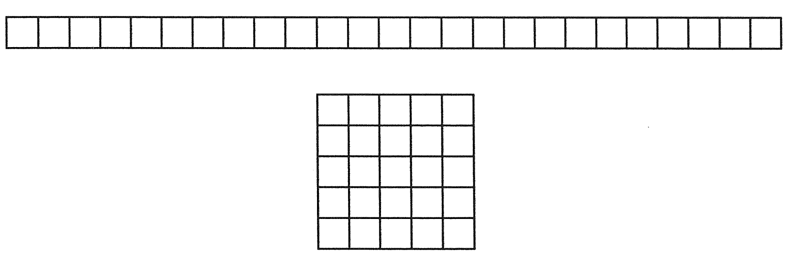
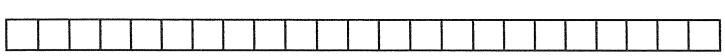


Figure 4



Investigation 3

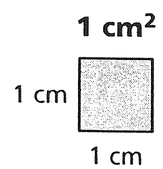
Measuring Triangles

You can find the area of a figure by drawing it on a grid (or covering it with a transparent grid) and counting squares, but this can be very time consuming. In Investigation 1, you found a rule for finding the area of a rectangle without counting squares. In this investigation, you will look for rules for finding the area of triangles using what you know about rectangles.

3.1 Triangles on Grids

Getting Ready for Problem 3.1

A square centimeter is 1 centimeter by 1 centimeter. It has an area of 1 square centimeter. Sketch a square centimeter such as the one here.



- Draw one diagonal to form two triangles.
- What is the area of each triangle?
- Is the perimeter of one of the triangles greater than, less than, or equal to 3 centimeters?

Problem 3.1 Finding Area and Perimeter

A. On the next page, six triangles labeled A–F are drawn on a centimeter grid.

1. Find the perimeter of each triangle.
2. Describe the strategies you used for finding the perimeters.
3. Find the area of each triangle.
4. Describe the strategies you used for finding the areas.

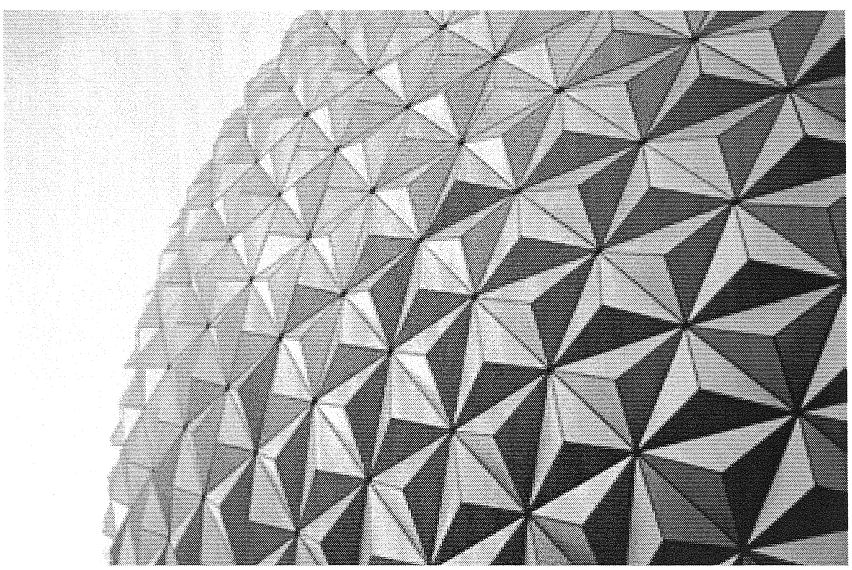
B. Look at triangles A–F again. Draw the smallest possible rectangle on the grid lines around each triangle.

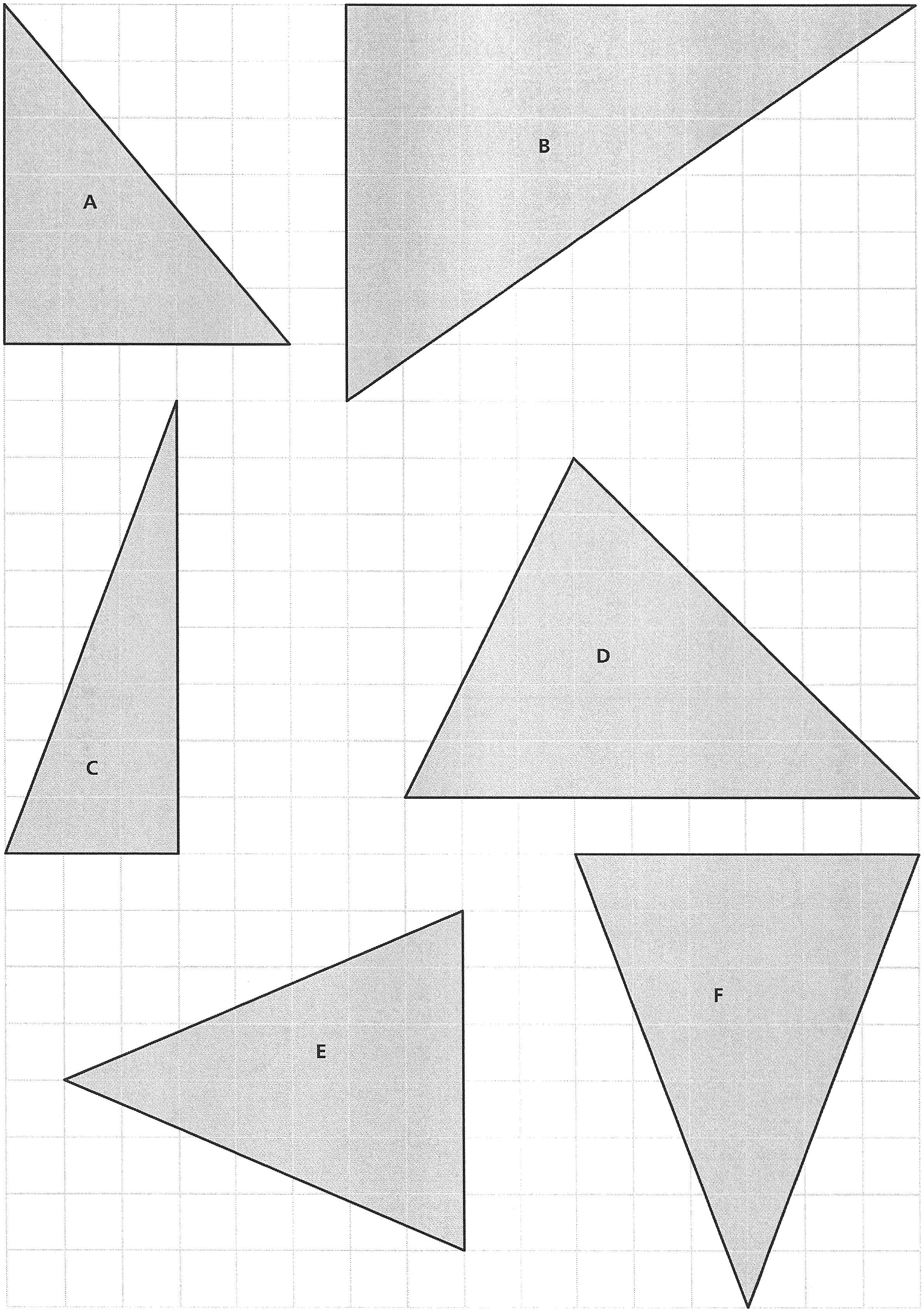
1. Find the area of each rectangle. Record your data in a table with columns labeled for the triangle name, the area of the rectangle, and the area of the triangle.
2. Use the data in your table. Compare the area of the rectangle and the area of the triangle. Describe a pattern that tells how the two are related.

C. Use your results from Question B. Write a rule to find the area of a triangle.

ACE **Homework starts on page 44.**

active math online
For: Areas and Perimeters of Shapes and Images Activity
Visit: PHSchool.com
Web Code: amd-5303




A
B
C
D
E
F

3.1 Triangles on Grids

Goals

- Develop and employ reasonable strategies for finding the area of a triangle
- Find relationships between rectangles and triangles
- Use these relationships to develop techniques for finding the area of a triangle

This section may take $1\frac{1}{2}$ days. The first day will use a complete Launch-Explore-Summarize sequence for Question A and launching Question B. In Question A, students will find the area and perimeter of six different triangles. Developing and using the formula for the area of a triangle is not the goal.

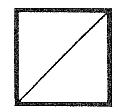
Next the students will use the same six triangles to complete Questions B and C. This time they will explore the relationship between area of a triangle and area of the smallest rectangle that surrounds it. Students will work with right and acute triangles in this problem. At the end of Problem 3.1, students should be able to verbalize that the area of a triangle is half the area of a rectangle when the rectangle is the smallest possible rectangle that can surround the triangle. This will be a first look at the relationship between the area of a rectangle and the area of a triangle. This relationship along with base, height, orientation, and obtuse triangles will be explored in the other problems in this investigation.

Day 1: Question A

Launch 3.1

Begin the problem by using the Getting Ready to introduce students to cm^2 notation. This notation will be used in future units.

Have students lay their paper over the diagram and copy the centimeter square, or give them a piece of centimeter grid paper. Talk about each question in the Getting Ready. The first question should not pose much difficulty. It suggests to students that a square can be divided into two congruent triangles.



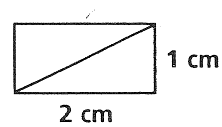
This idea is developed throughout this problem. Spend most of your time focusing on the second question.

Suggested Questions

- *What is the area of each triangle?* (The area of each triangle is $\frac{1}{2}$ cm^2.)
- *Would the perimeter of one of the triangles be greater than, less than, or equal to 3 cm?* (The perimeter is greater than 3 cm. The diagonal is longer than 1 cm, so the perimeter of each triangle is longer than 3 cm.
  1 + 1 + more than 1 = more than 3.)

The above question provides an opportunity to address a common mistake students make when finding the length of the diagonal side of a right triangle. Students often mistake the length of the diagonal to be the same as the other sides. By having students measure the diagonal of this triangle, they will discover that the diagonal is longer than either of the two sides that form the right angle. In this case, the diagonal is longer than 1 cm. This makes the perimeter of the triangle greater than 3 cm. Since students will be measuring the lengths with a centimeter ruler, be sure they know how to measure accurately to the nearest millimeter.

If you feel students need another example, have them draw a rectangle that is two square centimeters, insert the diagonal, and answer the two Getting Ready questions. Also discuss labels for area and perimeter using the cm^2 notation for area.



INVESTIGATION 3

Explain to students that they will be working on Question A only and then stopping to discuss their findings. Explain to the students that they will be asked to find the perimeters and areas of six triangles that are not covered with whole squares. Because students have already counted to find the areas of rectangles and then developed a formula, they might ask you to tell them the formula for triangles. Suggest that they think of strategies for finding the area of triangles while they work on the problems in this Investigation. Encourage students to look for patterns that would lead to a formula, as they did with rectangles.

Give students a copy of Labsheet 3.1. Have students work in pairs to find the perimeters and areas of the triangles.

Day 1: Question A

Explore 3.1

As you circulate, remind students to record their findings and explain how they arrived at their answers.

For Question A, part (1), be sure that students are measuring the length of the diagonal (or slanted) sides of triangles with a centimeter ruler so that they get an accurate measure for perimeter. Students can also mark the length of a slanted side on the edge of a sheet of paper and compare it to the centimeter grid on which the triangles are drawn. If students have perimeters that seem unreasonable, challenge them to measure again and compare. Check to see if students are lining up the ruler correctly and reading the measurement accurately.

Day 1: Question A

Summarize 3.1

Ask students for the measures they found for each figure, and record their answers on the board. Continue to collect answers (even if you have several different areas proposed for the same figure) until all the answers that groups found are recorded. Transparency 3.1C is provided for students to come up to the overhead projector and show their work.

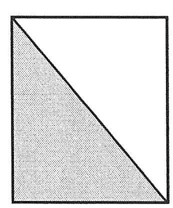
Figure	Perimeter (cm)	Area (cm^2)
A	about 18.8	15
B	about 29.2	35
C	about 19.5	12
D	about 24.2	27
E	about 21.2	21
F	about 23	24

Suggested Questions Focus the class's attention on the chart.

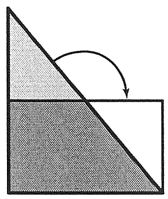
- *How did you find these perimeter measurements?* (At this point, most students will understand that perimeter is the length around a figure and that they need to measure the three edges of a triangle and add the results. If there are disagreements about the perimeter, you might want to add three more columns to your table, collect the measures of the lengths of the sides, and resolve the perimeter differences. If other students agree with the side lengths reported, then perhaps students totaled incorrectly. Have students come up to the overhead and measure the side lengths where there is disagreement.)
- *How did you find these area measurements?* (Here are three possible ways that students might find the areas of the triangles in Question A.

  Students may count the number of whole square centimeters and then estimate how many partial square centimeters there are.

  Students may enclose the triangle in a rectangle twice as large as the triangle, find the area of the rectangle, and then divide that by 2.



Students may chop off half of the height of the triangle and rearrange it to make a rectangle for which they can find the area. Some students describe this as making a rectangle as wide as the triangle but with half the height.



In the case of triangles D–F, they may enclose the triangle in a rectangle, find the area of the rectangle, and subtract the area of the two corner right triangles. They may also subdivide the triangles into rectangles and right triangles.)

Suggested Questions As students offer their methods for finding the areas of the triangles, ask other students if they agree with the methods.

- *Do you agree with what Tinesha did? Why?*
- *Did anyone use another approach?*
- *How did you find the area?* (Students will likely have found the area by counting. Ask whether they tried any other strategies. Again, we are not looking for the formula for finding the area of a triangle at this point, but the following questions could be used to start students thinking about other strategies.)
- *Which of the triangles are right triangles?* (triangles A, B, and C)
- *How do you think right triangles are related to rectangles?*
- *How could you get a right triangle from a rectangle?*

If students have no ideas, demonstrate with two sheets of paper that are the same size. Fold one sheet on the diagonal, and cut along the fold.

- *What shapes do I have now?* (two right triangles)
- *How do the two right triangles compare in size and shape?* (Lay one triangle on top of the other to show that they are the same size and shape.)
- *What measures of the right triangles are the same as measures of the original rectangle?*

Hold up the uncut sheet of paper so students can compare the two figures. They should notice that the length and width of the original rectangle are now two edges of the right triangle.

- *Will the areas of the rectangle and the right triangle be the same?* (no)
- *How do they compare?* (The right triangle is half the original rectangle, so the area of the right triangle is half the area of the rectangle.)
- *How might knowing this relationship help you to find the area of any right triangle?* (Some students may be able to verbalize that the area of a right triangle is half the area of a rectangle whose length and width are equal to the lengths of the two sides of the triangle that form the right angle. It is not important that all students be able to verbalize this relationship at this time. You just want them to think about how their knowledge of rectangles might help them to find the area of triangles.)

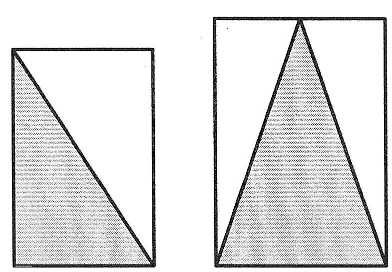
Use the relationship between a right triangle and a rectangle to launch Questions B and C.

Day 1: Questions B and C

Launch 3.1

Students will need Labsheet 3.1, which they completed when working on Question A. They will also need a new copy of Labsheet 3.1.

Read Questions B and C. Make sure students understand that they need the smallest rectangle that fits around the triangle. You might want to draw a right triangle and an isosceles triangle and talk about how the smallest rectangle will have the same base and height as the triangle.



Students can continue to work in pairs.

INVESTIGATION 3

Day 2: Questions B and C

Explore 3.1

As students work, check to be sure they are using the smallest rectangles. Listen to how students are describing the relationship between the areas of the rectangle and the areas of the triangles.

Suggested Question For students who finish quickly and can correctly identify the relationship between the triangle and the smallest enclosing rectangle, ask:

- *Does this relationship occur with perimeter?* (No. This is because at most two of the sides of the rectangle are also sides of the triangle. There is at least one additional side, of a different length, in the triangle.)

Day 2: Questions B and C

Summarize 3.1

Ask students for the measures they recorded in their table and record them in a table at the board. Transparency 3.1D is provided for students to come up to the overhead projector and show their work. If students offer two different answers for a problem, stop and talk about how they arrived at those answers.

Design	Area of Rectangle (cm^2)	Area of Triangle (cm^2)
A	30	15
B	70	35
C	24	12
D	54	27
E	42	21
F	48	24

Suggested Questions Talk about the comparisons students made when they compared the areas of the rectangles and triangles.

- *How are the areas of the triangle and smallest enclosing rectangle related?* (The area of the smallest rectangle is twice the area of the triangle.)
- *Will the perimeter of the smallest rectangle be twice the length of the triangle's perimeter?* (No.)

Move to Question C.

- *How could we write a rule to find the area of a triangle?*

At this point students can offer a rule using words. For example, "If you find the area of the smallest enclosing rectangle and divide it by 2, you will get the area of the triangle."

Help them connect their rule for a triangle to the formula they developed in Investigation 1 for area of a rectangle. For example, with Triangle A on Labsheet 3.1,

- *So, if you multiply the length and the width of the smallest enclosing rectangle, you will get the area of the rectangle?* (Yes)
- *How are the measurements for length and width of the rectangle related to the triangle?* (They are as high and as wide as the triangle.)

Using these words, you would find the area of a triangle by multiplying the length and width of the smallest enclosing rectangle and dividing by two or $(l \times w) \div 2$. Talk briefly with them about how you do not use the labels *length* and *width*.

- *In a triangle these parts are called base and height. In the next problem we will learn about base and height, and modify our rule for finding area of a triangle to be more like the one that mathematicians use.*

3.1 Triangles on Grids (Day 1)

At a Glance

PACING 1 day

Mathematical Goals

- Develop and employ reasonable strategies for finding the area of a triangle
- Find relationships between rectangles and triangles

Launch

Materials
- Transparencies 3.1A, 3.1B

Question A

Use the Getting Ready to introduce students to cm^2 notation. Have students lay their paper over the diagram and copy the centimeter square. Talk about each question.

If you feel students need another example, have them draw a rectangle that is 2 square centimeters, insert the diagonal, and answer the two Getting Ready questions. Also discuss labels for area and perimeter using the cm^2 notation for area.

Explain to students that they will be working on Question A only and then stopping to discuss their findings. Suggest that they think of strategies for finding the area of triangles while they work on the problems in this Investigation. Encourage students to look for patterns that would lead to a formula, as they did with rectangles.

Have students work in pairs.

Explore

Materials
- Labsheet 3.1

Question A

Remind students to record their findings and explain how they arrived at their answers.

Be sure that students measure the length of the diagonal sides of triangles with a ruler so that they get an accurate measure for perimeter. Check to see if students line up the ruler correctly and read the measurement accurately.

Summarize

Materials
- Transparency 3.1C
- Student notebooks

Question A

Record students' answers on the board. Focus the class's attention on the chart.

- *How did you measure the perimeter?*

Have students come up to the overhead and measure the side lengths where there is disagreement.

- *How did you measure the area?*

continued on next page

Summarize
continued

As students explain their methods for finding the areas of the triangles, ask other students if they agree with the methods.

- *Which of the triangles are right triangles?*
- *How do you think right triangles are related to rectangles?*
- *How could you get a right triangle from a rectangle?*

Ask students to think about how their knowledge of rectangles might help them to find the area of triangles.

Launch

Questions B and C

Students will need Labsheet 3.1 from Question A. They may also need a new copy of Labsheet 3.1.

Read Questions B and C. Make sure students understand that the smallest rectangle is supposed to fit around the triangle. You might want to draw a right triangle and an isosceles triangle and talk about how the smallest rectangle will have the same base and height as the triangle.

Materials
- Transparency 3.1B
- Labsheet 3.1 (one fresh copy per student, optional)

ACE Assignment Guide for Problem 3.1 (Day 1)

Differentiated Instruction

Core 1–6
Labsheet 3ACE Exercises 1–6 is provided if Exercises 1–6 are assigned.

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 3.1 (Day 1)

A. 1. Triangle A: about 18.8 cm
Triangle B: about 29.2 cm
Triangle C: about 19.5 cm
Triangle D: about 24.2 cm
Triangle E: about 21.2 cm
Triangle F: about 23 cm

2. Strategies will vary but should include that the length of a diagonal side was measured with a centimeter ruler.

3. Triangle A: 15 cm^2
Triangle B: 35 cm^2
Triangle C: 12 cm^2
Triangle D: 27 cm^2
Triangle E: 21 cm^2
Triangle F: 24 cm^2

4. Strategies will vary but may include cutting and rearranging the triangle into a rectangle, or surrounding a triangle by a rectangle, then finding the area of the rectangle and dividing it by 2.

3.1 Triangles on Grids (Day 2)

At a Glance

PACING $\frac{1}{2}$ day

Mathematical Goals

- Find relationships between rectangles and triangles
- Use these relationships to develop techniques for finding the area of a triangle

Explore

Questions B and C

As students work, check to be sure they are using the smallest rectangles. Listen to how students are describing the relationship between the areas of the rectangle and the areas of the triangles.

For students who finish quickly and can correctly identify the relationship between the triangle and the smallest enclosing rectangle, ask:

- *Is the same relationship true with perimeter?*

Summarize

Questions B and C

Record students' measures in a table at the board. Discuss how they arrived at those answers.

- *How are the areas of the triangle and smallest rectangle related?*
- *Will the perimeter of the smallest enclosing rectangle be twice the length of the triangle's perimeter?*

Move to Question C.

- *How could we write a rule to find the area of a triangle?*

Help students connect their rule for a triangle to the formula they developed in Investigation 1 for area of a rectangle.

- *How are the measurements for length and width of the rectangle related to the triangle?*

Talk briefly about how triangles do not use the labels *length* and *width*, but rather *base* and *height*.

Materials

- Transparencies 3.1C, 3.1D
- Student notebooks

Vocabulary

- base
- height

ACE Assignment Guide for Problem 3.1 (Day 2)

Differentiated Instruction
Solutions for All Learners

Core 26–31
Labsheet 3ACE Exercises 26–31 is provided if Exercises 26–31 are assigned.

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 3.1 (Day 2)

B. 1.

Design	Area of Rectangle (cm²)	Area of Triangle (cm²)
A	30	15
B	70	35
C	24	12
D	54	27
E	42	21
F	48	24

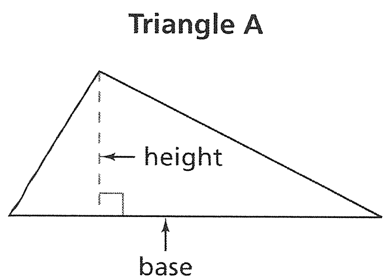
2. The area of a rectangle is twice the area of a triangle or the area of the triangle is half the area of the rectangle.

C. Possible answer: $(b \times h) \div 2$ (NOTE: $(\ell \times w) \div 2$ is acceptable at this time.)

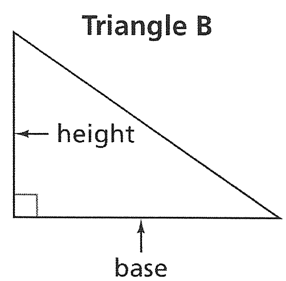
3.2 More Triangles

Base and height are two words that describe triangles. The **base** of a triangle can be any one of the sides of the triangle. "Base" also refers to the length of the side you choose as the base. The **height** of a triangle is the perpendicular distance from the top vertex to the base.

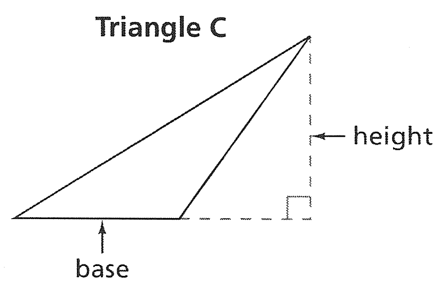
You can think of the height of a triangle as the distance a rock would fall if you dropped it from the top vertex of the triangle straight down to the line that the base is on.



Measure the height inside the triangle.



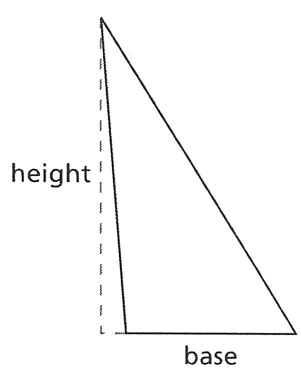
Measure the height along one of the triangle's sides.



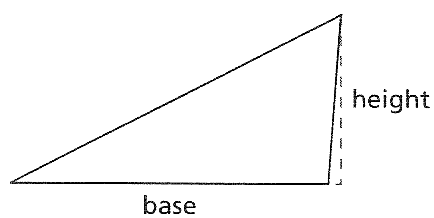
Measure the height outside the triangle.

The side you identify as the base also determines what the height is.

Look at triangle A again. Suppose you turn triangle A so it rests on its shortest side. The shortest side of the triangle becomes the base. The height is measured outside and to the left of the triangle.



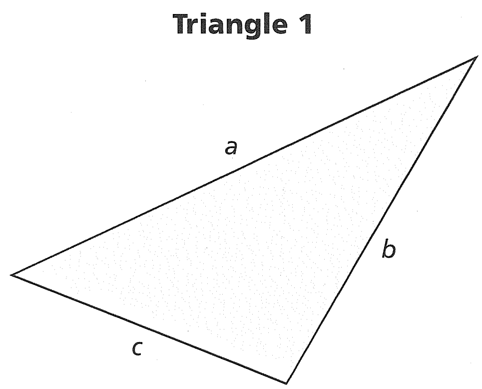
Suppose you turn triangle A again. The second longest side becomes the base. The height is measured outside and to the right of the triangle.

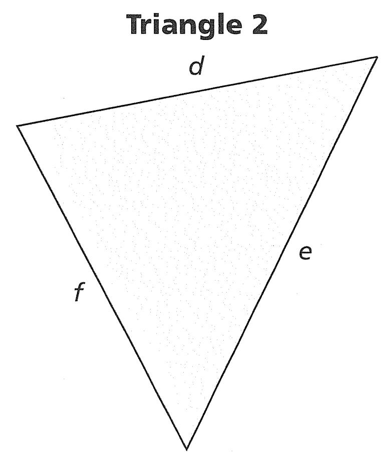


In this problem, you are going to explore how changing the position or orientation of a triangle affects the base, height, and area of a triangle.

Problem 3.2 Identifying Base and Height

A. Cut out copies of Triangles 1 and 2. Position each triangle on centimeter grid paper.





1. Label the base and height of each triangle.
2. Find the area of each triangle. Explain how you found the area. Include any calculations that you do.

B. Find a second way to place each triangle on the grid paper.

1. Label the base and height of each triangle in its new position.
2. Find the area of each triangle. Explain how you found the area. Include any calculations that you do.

C. Does changing which side you label the base change the area of the triangle? Explain.

D. When finding the area of a triangle, are there advantages or disadvantages to choosing a particular base and its corresponding height? Explain.

ACE **Homework starts on page 44.**

3.2 More Triangles

Goals

- Distinguish among base, height, and side lengths of triangles
- Understand that depending upon how you position a given triangle, it has more than one base and height, but only one area

In this section students are introduced to the conventions of base and height as they apply to triangles. Using these conventions, they will explore how changing the position or orientation of a triangle affects the base, height, and area of a triangle. This exploration will also lead to an efficient rule for finding the area of any triangle.

When working with area and perimeter of triangles, there are some important differences from rectangles. In particular, the terms *length* and *width* are ambiguous for triangles. The length of a rectangle is always the length of one of the rectangle's sides and likewise for the width. For a triangle, the base is the length of one of the triangle's sides, but the height is not necessarily the same length as a side. When calculating area of a triangle, the product of the base and height is divided by 2 because a triangle has half the number of square units that a rectangle with corresponding base and height (length and width) has.

Launch 3.2

Discuss the overview of base and height in the introduction in the student edition. The *height* of a triangle is the perpendicular distance from the vertex to the base. In the first triangle shown, the height falls inside the figure. In the right triangle, the height is one of the sides. For the third triangle, the height falls outside the figure.

You may want to use the triangles from Problem 3.1 to help students understand base and height. Place Transparency 3.1A of the triangles on the overhead projector.

Suggested Question Ask:

- *What are the base and height of Triangle A?*

If someone tells you 6 units for height and 5 units for base (which is correct), ask him or her to explain why. If no one figures this out, explain that height is the perpendicular distance from the side you have identified as the base to the vertex opposite the base.

Check for understanding by having students find the base and height of some other triangles. Be sure to have students identify side lengths on triangles where the sides are not the height (e.g., Triangle D on Labsheet 3.1).

Suggested Questions Return to the ideas raised at the end of the second summary discussion of Problem 3.1.

- *How do the base and height identified for each triangle correspond to the length and width of the rectangles you used to surround the triangles in Problem 3.1?* (Help students see that when they multiply the length and width of the smallest enclosing rectangle, this is numerically equivalent to multiplying the base and the height on the triangle. Talk briefly with students about why triangles do not use the labels *length* and *width* and what base and height measure.)
- *How do your words for describing the method of finding area relate to the symbolic representation for area of a triangle?* [Using their words, to find the area of a triangle, you multiply the length (number of square units in one row) and width (the number of rows) of the smallest enclosing rectangle and divide by two, or $(\ell \times w) \div 2$. With a triangle, these parts are called the base and the height. This leads to the formula $(b \times h) \div 2$.]
- *How does the height of a triangle relate to the height of a rectangle?* (The height of the triangle is the same as the width of the rectangle. Note that the height of the triangle may not be the length of a side of the triangle. For example, the height of Triangle D on Labsheet 3.1 is inside the triangle, but it has the same measure as the height of the rectangle surrounding it.)

Next, discuss the second part of the introduction on orientation. As you discuss how orientation changes where the base and height fall, it helps to model what students will be doing in the problem. In the problem, they will cut out a triangle, place it on grid paper (aligning a side with the grid), trace it, and label the base and height. They will then place the same triangle in a different position on grid paper (aligning a side with the grid), trace it, and label the base and height.

Now read the problem with the students and be sure they understand what they are to do. Be sure to read Questions C and D pointing out that they should think about these two questions while they work on Questions A and B. Provide students with Labsheet 3.2A, centimeter grid paper, and centimeter rulers to use to make accurate measurements for the base and height.

Have students work in pairs, each making his or her own drawings.

Explore 3.2

Encourage each pair to compare their measurements and discuss how to approach the problems. Students need to be careful when they cut out and trace their triangles onto grid paper.

If students are way off in their measurements, check to see if their drawings are accurate and whether they are using the ruler or counting the grid correctly.

As you circulate, remind students to label their drawings so that they are easy to refer to during the summary. If you are short on time, start the summary once students have finished Questions A and B and discuss Questions C and D as a class.

Meeting Special Needs

Labsheet 3.2B has several shaded copies of each triangle. These are provided so that you can copy them onto a transparency and cut them out for students who struggle to transfer the triangles onto grid paper or to visualize the grid beneath them. The shading on this labsheet is light enough to allow the grid lines to show through.

Summarize 3.2

Have a student or student pair come up to the overhead projector to show how they placed Triangle 1 on grid paper, where they think base and height are, and how they found the area.

Suggested Questions Ask other students who used this orientation if they used a different approach to find the area.

- *Did anyone find the area of this triangle a different way?*

Once you have discussed various ways to find the area of the triangle in one orientation, look at a second orientation of Triangle 1. Have another student or student pair come up and show how they placed the triangle, where they think the base and height are, and how they found the area. Again ask:

- *Did anyone use a different orientation or place the triangle on the grid a different way?*

Even though the problem asks students to use two orientations, there are three possible orientations that can emerge across the work the class does. As students present the different orientations, make a table to keep track of the base, height, and area.

Triangle 1

Base (cm)	Height (cm)	Area (cm^2)
7 (longest side)	3	$10\frac{1}{2}$
4 (shortest side)	$\approx 5\frac{1}{4}$	$\approx 10\frac{1}{2}$
$\approx 5\frac{1}{4}$ (other side)	4	$\approx 10\frac{1}{2}$

Ask questions that prompt students to talk about base and height in terms of what is being measured. When 7 cm is given for the base, ask students what the 7 cm represents.

- *What is the 7 cm you found for the base a measure of?* (the length of the horizontal side of the triangle)
- *What does the 3 cm you found for height represent?* (the vertical distance between the base and the vertex of the triangle)

Once all three orientations are presented ask Question C:

- *Does changing the base of the triangle change the area of the triangle? (No) Why? (The shape does not change, just the position in which it is placed.)*
- *If the measurements are slightly different, ask why.* (measurement error)
- *How can there be three different ways to find the same area?* (The area does not change when the shape is turned. Since there are three sides of a triangle, there are potentially three bases. Each base is associated with a unique height.)

Have students present the measurements they collected for Triangle 2. Since Triangle 2 is an isosceles triangle, there are only two different sets of measurements.

Triangle 2

Base (cm)	Height (cm)	Area (cm^2)
6 (longest side)	4	12
5 (shortest side)	$\approx 4\frac{4}{5}$	≈ 12

- *Why does this triangle have two sets of measurements when Triangle 1 had three?* (It is an isosceles triangle. Two of the sides (bases) are the same length and they each have the same corresponding height measurements.)

If you have time it would be interesting to use Shape T, a right triangle, from the Shapes Set or any cut-out right triangle. With right triangles, the base and height can fall on the sides of the triangle. Rather than measure, show students the triangle and tell them the short side is 5 cm long. Give a chart with the base, height, and area measurements and ask students to tell you which way to place the triangle onto the centimeter grid paper.

Base (cm)	Height (cm)	Area (cm^2)
$\approx 9\frac{1}{2}$ (longest side)	$\approx 4\frac{1}{4}$	$\approx 20\frac{3}{16}$
5 (shortest side)	8	20
8 (other side)	5	20

Suggested Questions Discuss Question D.

- *Are there advantages or disadvantages to choosing a particular base and its corresponding height to find the area of a triangle?*

Students may offer different views. There is no "best" answer to this problem. Ask them about the different types of triangles (right, isosceles, obtuse, equilateral). Students might offer the following ideas:

It is easier to find the area when the height falls inside the rectangle.

It is easier to find the position that gives the nicest measurements, such as whole-number measurements.

With a right triangle it is easier to use the position where the sides are the base and height.

With an obtuse triangle (like Triangle 1) it may be easier to turn the triangle so the vertex is above the base.

At a Glance

3.2 More Triangles

PACING 1 day

Mathematical Goals

- Distinguish among base, height, and side lengths of triangles
- Understand that depending upon how you position a given triangle, it has more than one base and height, but only one area

Launch

Discuss the overview of base and height in the introduction in the Student Edition. Check for understanding by having students find the base and height of some other triangles.

Return to the ideas raised at the end of Problem 3.1. Talk about how the base and height they identify for each triangle correspond to the length and width of the rectangles they used to enclose the triangles in Problem 3.1.

Help students understand how their words are related to the symbolic representation for area of a triangle.

Discuss the second part of the introduction on orientation. Model what students will do in the problem.

Read the problem with the students. Have students work in pairs.

Materials

- Transparencies 3.1A, 3.2A, 3.2B
- Labsheet 3.2A (one per student)
- scissors
- cut-outs from a transparency copy of Labsheet 3.2B (optional—see Explore section)

Explore

Pairs can compare their measurements and discuss how to approach the problems.

Check students' drawings and measurements as they work. Remind students to label their drawings so that they are easy to refer to during the summary.

Summarize

Discuss the various ways to orient Triangle 1, as well as the area calculations that result from each.

- *Did anyone find the area of this triangle a different way?*
- *Did anyone use a different orientation or place the triangle on the grid a different way?*

As students present the different orientations, make a table to keep track of the base, height, and area. Once all three orientations are presented, ask Question C:

- *Does changing the base of the triangle change the area of the triangle?*
- *Why not? Why are these three triangle area measurements slightly different?*
- *How can there be three different ways to find the same area?*

Materials

- Student notebooks

continued on next page

Summarize *continued*

Repeat with Triangle 2. If you have time, use Shape T from the Shapes Set to repeat the conversation.

Finish by discussing the advantages to different orientations.

- *Are there advantages and disadvantages to choosing a particular base and its corresponding height to find the area of a triangle?*

ACE Assignment Guide for Problem 3.2

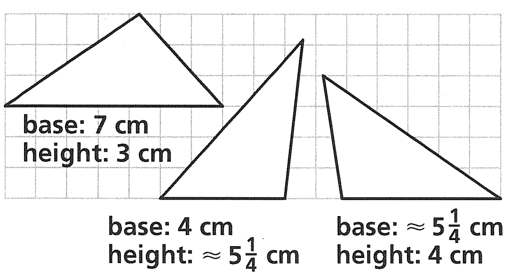
Core 7–10, 13–17

Other *Applications* 11, 12, 18–20; *Connections* 32–34; unassigned choices from previous problems

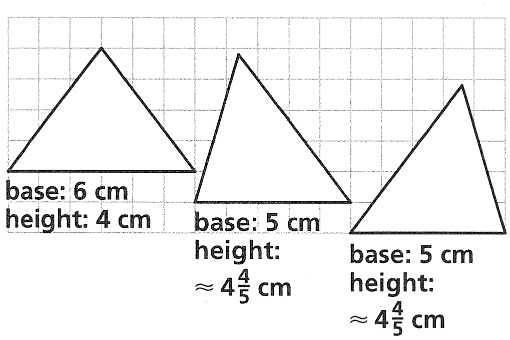
Adapted For suggestions about adapting Exercise 17 and other ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 3.2

A. 1. Possible choices for positioning and labeling Triangle 1 shown below.



Possible choices for positioning and labeling Triangle 2 are shown below.



2. The following are based on approximate measurements. This is why the area of each triangle varies. Some students may count the squares. Some may imbed it in a rectangle and subtract off excess area. Some may use the height-base relationship.

Triangle 1

Base (cm)	Height (cm)	Area (cm^2)
7 (longest side)	3	$10\frac{1}{2}$
4 (shortest side)	$\approx 5\frac{1}{4}$	$\approx 10\frac{1}{2}$
$\approx 5\frac{1}{4}$(other side)	4	$\approx 10\frac{1}{2}$

Triangle 2

Base (cm)	Height (cm)	Area (cm^2)
6 (longest side)	4	12
5 (shortest side)	$\approx 4\frac{4}{5}$	≈ 12
5 (other side)	$\approx 4\frac{4}{5}$	≈ 12

B. 1. See answers to Question A, part (1).

2. See answers to Question A, part (2).

C. No, the area of a shape does not change when it is repositioned. The base and height measurements may change but they will still lead to the same area. However, the area we calculate might be slightly different because of the approximations we make when we measure the base and height.

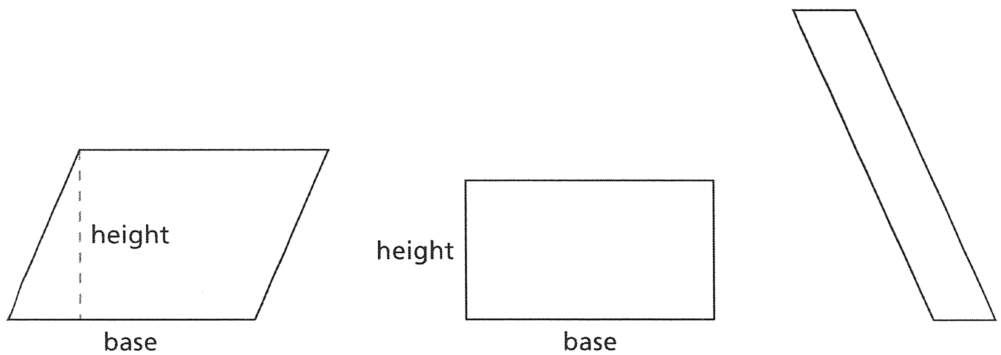
D. Students' answers may vary. Here are two possibilities. See summary for others.

Possibility 1: It is easier to find the area when the height falls inside the rectangle.

Possibility 2: It is easier to find the position that gives the nicest measurements, such as whole-number measurements.

Getting Ready for Problem 4.1

Here are three parallelograms with the base and height of two parallelograms marked. What do you think the *base* and the *height* of a parallelogram mean? How do you mark and measure the base and height of the third figure?



Problem 4.1 Finding Measures of Parallelograms

Six parallelograms labeled A–F are drawn on the centimeter grid on the next page.

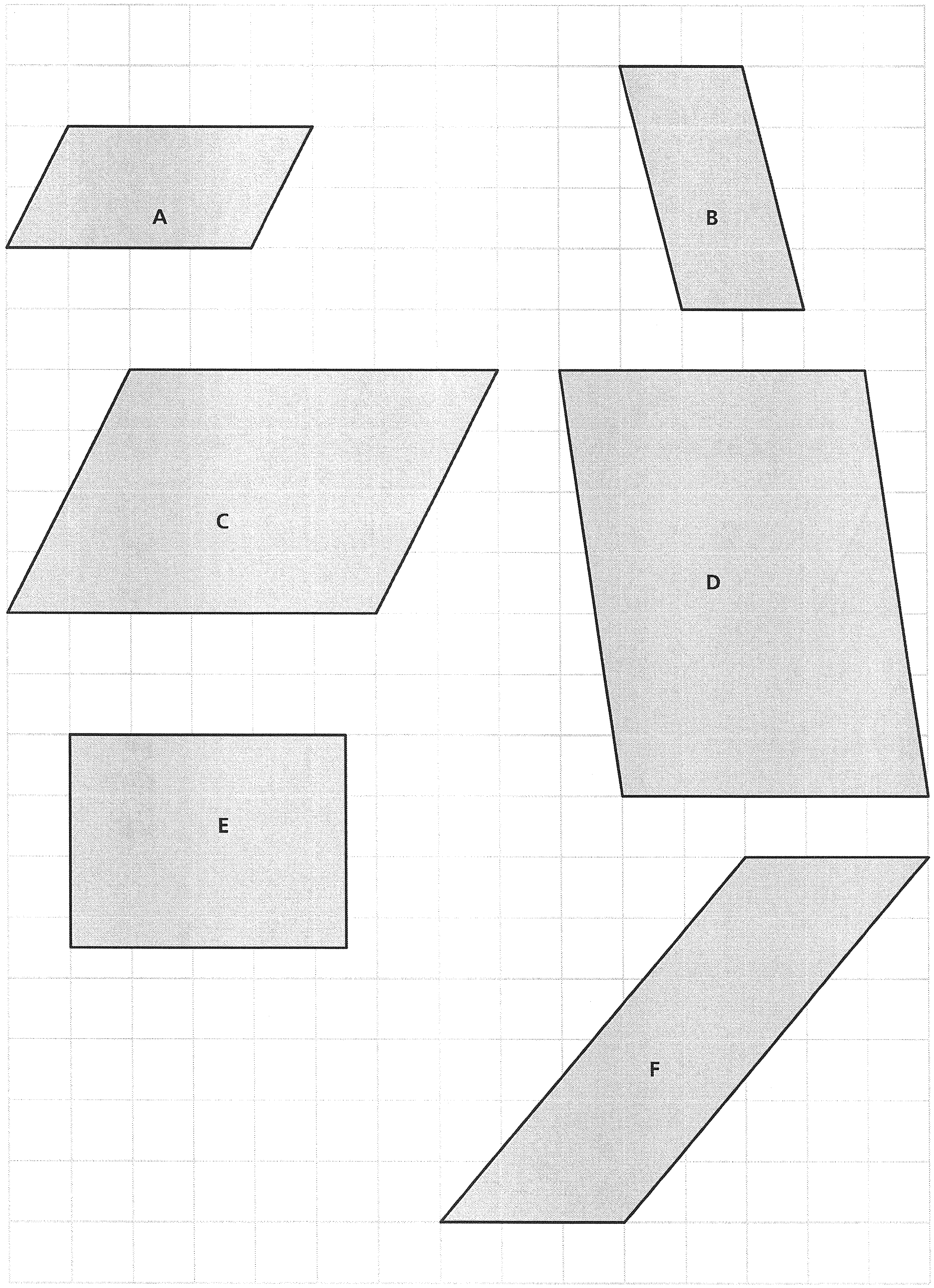
A. **1.** Find the perimeter of each parallelogram.

2. Describe a strategy for finding the perimeter of a parallelogram.

B. **1.** Find the area of each parallelogram.

2. Describe the strategies you used to find the areas.

ACE **Homework starts on page 60.**


A
B
C
D
E
F

4.1 Finding Measures of Parallelograms

Goal

- Develop and employ reasonable strategies for finding the areas and perimeters of parallelograms

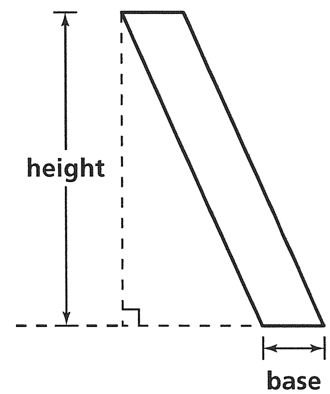
Students will find the area and perimeter of six different parallelograms. The purpose is to have students develop and employ reasonable strategies for finding the area of these parallelograms: counting, estimating, and talking about ways to rearrange and use parts of grid squares.

Launch 4.1

Begin by discussing the Getting Ready in the introduction to Problem 4.1. Rather than show students where the base and height are on the third parallelogram, it is left open to see if students can apply what they know about bases and heights of triangles to parallelograms.

Suggested Questions

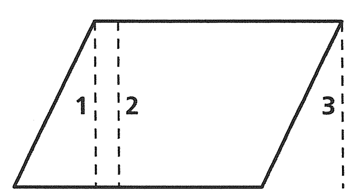
- *Where are the base and the height on a triangle?* (The base is the length of one of the sides of the figure and the height for that base is the perpendicular distance from the base to an opposite vertex.)
- *What can measures of the base and height tell you?* (Measures of the lengths of the base and height can tell you the number of square units in a row and the number of rows with that many square units. The base of a parallelogram is one side of the parallelogram. It could be any of the four sides. The height of a parallelogram is any perpendicular from the base to the side parallel to the base. The height of a parallelogram depends on the side that is chosen for the base.)



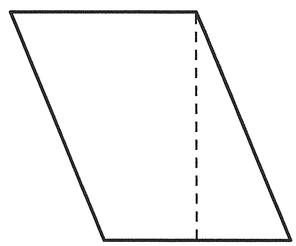
- *Do height and base mean the same as they did for triangle?* (The base and height have the same meaning when used with parallelograms as they did when used with triangles. You can think of the height as the distance a rock would fall if you dropped it from a point at the top of a parallelogram perpendicular down to the line the base is on.)

Have students come to the overhead projector and show where they think the base and height are on the parallelograms on Transparency 4.1B.

- *Where would the base and height be on each parallelogram?* (Unlike triangles, parallelograms have multiple places where the line representing the height for a given base can be positioned. Using the first parallelogram in the Getting Ready, the diagram below shows three possible places where the height can be drawn if the bottom side is used as the base.)



- *With triangles, you could label different bases and heights if you placed the triangle in different positions. How can I change the position of the parallelograms in the Getting Ready and label a different base and height for each one?* (Turn the parallelogram and use the short side for the base as shown in the diagram below.)



Read Questions A and B of Problem 4.1 to students. Provide them with a copy of Labsheet 4.1. Have students work in pairs.

Explore 4.1

As you circulate, remind students to record their findings and describe their strategies.

Be sure students are measuring the length of the diagonal sides of triangles with a centimeter ruler so that they get an accurate measure for perimeter. If students have perimeters that seem incorrect, have them measure a second time with a centimeter ruler. Check to see if students are lining up the ruler correctly and reading the measurement accurately.

Summarize 4.1

Ask students for the measures they found for each figure, and record their answers on the board.

Figure	Perimeter (cm)	Area (cm^2)
A	about $12\frac{1}{2}$	8
B	about $12\frac{1}{5}$	8
C	about 21	24
D	about 24	35
E	16	$15\frac{3}{4}$
F	about $21\frac{4}{5}$	18

Suggested Questions Focus the class's attention on the chart.

- *How did you find these perimeter measurements?* (At this point, most students will understand that perimeter is the length around a figure. Since parallelograms have opposite sides congruent, like rectangles, there are several formulas that can be used to find the perimeter. Possible answers include: add the lengths of the sides; add the lengths of the two sides that form an angle and double; or double one side length, double the other side length, and total.)

As students describe their methods, record them in words and ask:

- *How could I write this method for finding perimeter of a parallelogram as a rule with symbols?* (Example answer: perimeter of parallelogram = length of side a + length of side a + length of side b + length of side b or $P = a + a + b + b$.)

Now move to discussing area.

- *How did you find the area measurements for the parallelograms?* (Here are some possible ways that students might find the areas of the parallelograms:

  Students may count the number of whole square centimeters and then estimate how many partial square centimeters there are.

  Students may cut off part of the parallelogram and then rearrange the parts to form a rectangle, and find the area of the rectangle.

  Some students may notice that the area of the parallelograms in the table for Question A are approximately equal to the base times the height.

  Some students may draw a diagonal and use the two congruent triangles to find the area.

  If students offer the formula $b \times h$ for area, ask them to explain why this makes sense. If they cannot explain why the formula works, you can write it up on the board and put a question mark by it. Explain that this is an idea for the class to continue to think about.)

Remember, developing a formal rule for finding the area of a parallelogram is not the goal for this problem.

- *Parallelogram A and B have the same area, but different perimeters. Does this make sense?* (Two shapes can have the same area and different perimeters.)

Use this summary to launch the next problem.

4.1 Finding Measures of Parallelograms

At a Glance

PACING 1 day

Mathematical Goal

- Develop and employ reasonable strategies for finding the areas and perimeters of parallelograms

Launch

Discuss the Getting Ready in the introduction.

- *Where are the base and the height on a triangle?*
- *What can measures of the base and height tell you?*

Explain that base and height have the same meaning when used with parallelograms as when used with triangles.

- *With triangles, you could label different bases and heights on one triangle if you placed the triangle in different positions. How can I change the position of the parallelograms in the Getting Ready and label a different base and height for each one?*

Read Questions A and B of Problem 4.1 with the students. Provide a copy of Labsheet 4.1.

Have students work in pairs.

Materials

- Transparencies 4.1A and 4.1B
- Labsheet 4.1

Vocabulary

- base
- height

Explore

Remind students to record their findings and describe their strategies.

Check how students are measuring as they work.

Summarize

Ask students for the measures they found for each figure, and record their answers on the board. Focus the class's attention on the table.

- *How did you find these perimeter measurements?*

As students describe their methods, record them in words and ask:

- *How could I write this method for finding perimeter of a parallelogram as a rule with symbols?*

Now move to discussing area.

- *How did you find the area measurements for the parallelograms?*
- *Parallelograms A and B have the same area, but different perimeters. Does this make sense?*

Materials

- Student notebooks

ACE Assignment Guide for Problem 4.1

Core 1–8
Other *Connections* 32
Labsheet 4ACE Exercises 1–7 is provided if Exercises 1–7 are assigned.

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 32: *Bits and Pieces I*

Answers to Problem 4.1

A. 1. Figure A: about $12\frac{1}{2}$ cm
Figure B: about $12\frac{1}{5}$ cm
Figure C: about 21 cm
Figure D: about 24 cm
Figure E: 16 cm
Figure F: about $21\frac{4}{5}$ cm

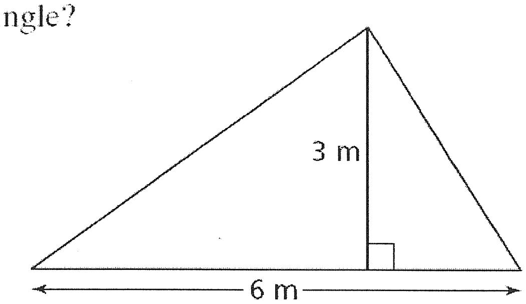
2. Possible answers include: add the lengths of the sides, add the lengths of the two sides that form an angle and double, or double one side length, double the other side length and total.

B. 1. Figure A: 8 cm^2
Figure B: 8 cm^2
Figure C: 24 cm^2
Figure D: 35 cm^2
Figure E: $15\frac{3}{4}$ cm^2
Figure F: 18 cm^2

2. Possible answers include: count the number of whole square centimeters and estimate how many partial square centimeters there are; or cut off part of the parallelogram by cutting perpendicular to the base, rearranging to make a rectangle, and then finding the area of the rectangle.

Covering and Surrounding Quiz -- Area and Perimeter of Triangles and Parallelograms
Name ______________________________ Period ____ Date ____________

1. What is the area of this triangle?



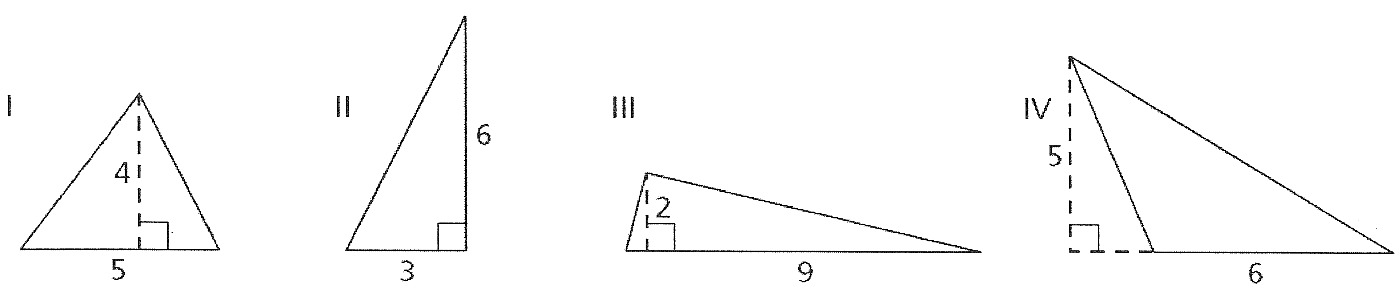
A. 18 sq. m B. 36 sq. m C. 9 sq. m D. 6 sq. m

2. Alexa wants to use ready-made 6-foot long fence sections for her yard. The yard is a rectangle with dimensions 30 feet by 36 feet. How many fence sections will she need to enclose her entire yard?

F. 22 G. 132 H. 66 J. 120

3.

Which two figures below have the same area?



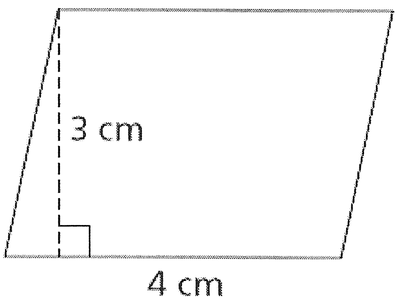
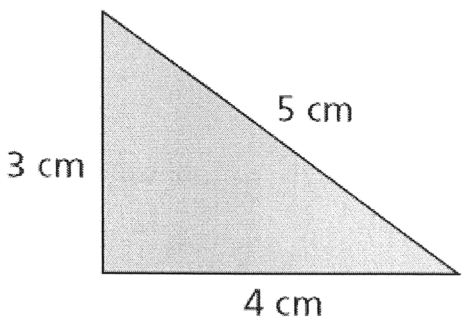
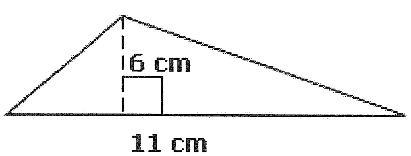
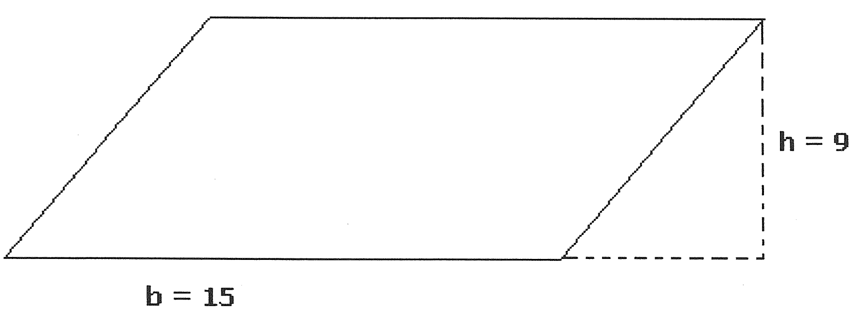
A. Figures I and II B. Figures II and III

C. Figures II and IV D. Figures I and IV

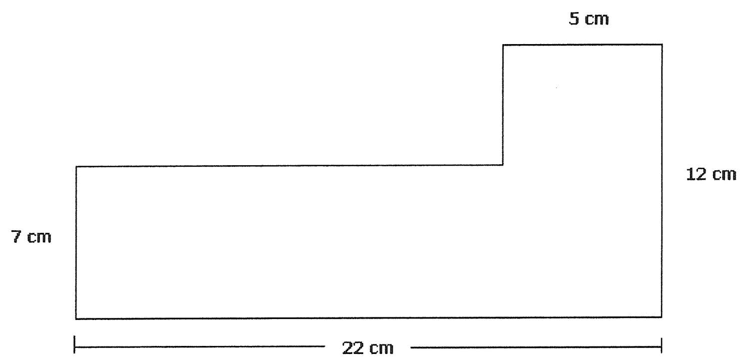
4. The area of a parallelogram is 45 square inches. The base of the parallelogram is 5 inches.

Find the height of the parallelogram? Draw a picture to help you.

5.

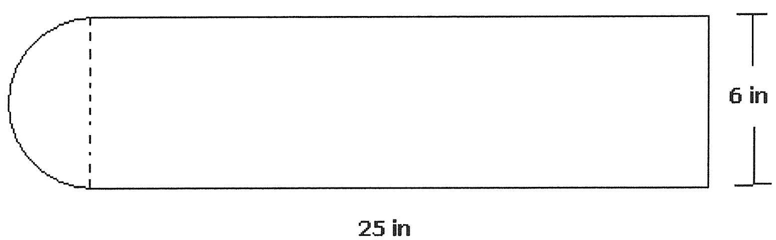
	Area =
	Area = Perimeter =
	Area =
	

Area and Perimeter of Composite Figures Name ____________________ Period_____



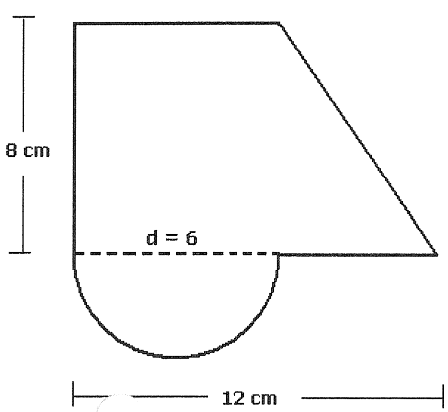
Find the area of this composite figure. First, find the area of the semi-circle, triangle, and square. Then, add each area together.

Find the perimeter of the composite figure to the nearest whole unit.



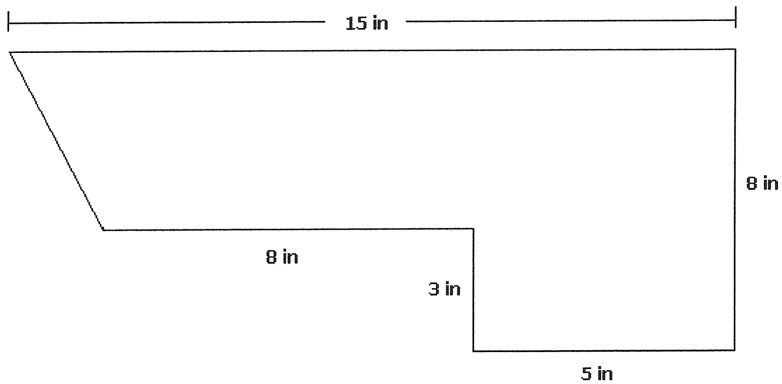
Find the area of the composite figure.

Find the perimeter of the composite figure to the nearest whole unit.



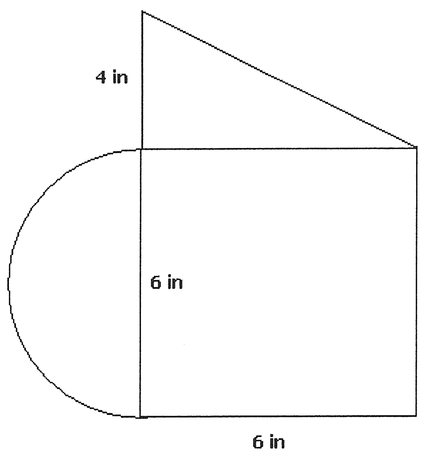
Find the area of the composite figure. .

Find the perimeter of the composite figure to the nearest whole unit.



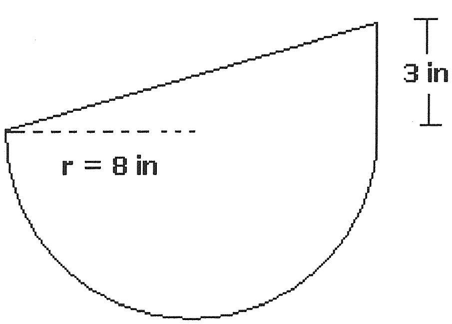
Find the area of the composite figure.

Find the perimeter of the composite figure to the nearest whole unit.



Find the area of the composite figure.

Find the perimeter of the composite figure to the nearest whole unit.



Find the area of the composite figure.

Find the perimeter of the composite figure to the nearest whole unit.

Name: ______________________________ Date: ______________ Period: ______

Area and Circumference of Circles Extension Problems

After Investigation 7 of Covering and Surrounding

You may use a calculator on this activity.

1. Determine the area of a circle with a diameter of 10 centimeters.

2. Determine the circumference of a circle with a radius of 4 inches.

3. Captain Cook determined that the distance around a circular island is 63 miles.

   A. Estimate the distance from the shore to the buried treasure in the center of the island?

   B. What is the area of the island?

4. Determine the circumference of a circle with a radius of 7 meters.

5. A circular cherry pie has a diameter of 11 inches and is cut into 8 equal size pieces.

- Draw a picture to model the situation.

- What is the approximate area of each piece of pie?

6. The circumference of a round table is approximately 188 inches.

   a. What is the distance from anywhere along the edge of the table to the center?

   b. What is the area of the table?

Filling & Wrapping Unit Test Grade 7

Name____________________________ Date__________________

Mathematics Formula Sheet for Grades 6-8

Figure	Formula	Variables
Circle	$A = \pi r^2$	A: Area r: radius
	$C = \pi d$ or $C = 2\pi r$	C: Circumference d: diameter r: radius
Cylinder	$SA = 2\pi r^2 + 2\pi rh$	SA: Surface Area r: radius h: height
	$V = \pi r^2 h$	V: Volume r: radius h: height
Cone	$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$	V: Volume r: radius h: height B: area of base
Rectangular Prism	$SA = 2lw + 2lh + 2wh$ or $SA = 2(lw + lh + wh)$	SA: Surface Area l: length w: width h: height
	$V = lwh$	V: Volume l: length w: width h: height
Pyramid	$V = \frac{1}{3}Bh$	V: Volume B: area of base h: height

Integers and Order of Operations Unit

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations
Comparing and Ordering Integers (*NO-i*) *** Be sure to include information on placing negative and positive integers on the number line.***	1	Online lesson		**6.2.D Apply the commutative, associative, and distributive properties and use the order of operations to evaluate mathematical expressions.** **6.5.B Locate positive and negative integers on the number line and use integers to represent quantities in various contexts.** **6.5.C Compare and order positive and negative integers using the number line, lists, and the symbols, <, >, =.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Lesson on Comparing and Ordering Integers and Fractions on the number line, lists, and with symbols	2			
Activity for Number Properties and Order of Operations: Order of Operations Bingo (meets 6.2.D)	2			
(CMP2) Accentuate the Negative Prob. 4.1 Order of Operations pg. 60	1			
(CMP2) Accentuate the Negative Prob. 4.2 Distributing Operations pg. 64	2			
Quiz on Integers and order of operations	1			
Total Instructional Days for Integer Introduction:	**9 days**			

Contents in Integers and Order of Operations Unit

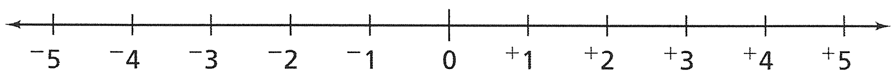
- Online lesson Topic 2: Understanding Integers
- Comparing and Ordering Fractions and Integers Practice worksheet
- Order of Operations Bingo
- CMP2 Accentuate the Negative: Investigation 4.1 SE
- CMP2 Accentuate the Negative: Investigation 4.1 TE
- CMP2 Accentuate the Negative: Investigation 4.2 SE
- CMP2 Accentuate the Negative: Investigation 4.2 TE
- Integer and Order of Operation Quiz

Topic 2: Understanding Integers

for use after ***Bits and Pieces I*** **Investigation 3**

Negative numbers are needed when the quantities are less than 0, such as very cold temperatures. Temperatures in winter can easily go below 0°F in some locations. An altitude of 0 feet is referred to as sea level, but there are places in the world that are below sea level.

The counting numbers and zero are called **whole numbers.** The first six whole numbers are 0, 1, 2, 3, 4, and 5. You can extend a number line to the left past zero.



The opposite of a positive number is a **negative number**. For example, the number $^{-}2$ is the **opposite** of $^{+}2$. The set of whole numbers and their opposites are called **integers**.

Problem 2.1

Emily, Sarah, Michael, Jacob, and Austin play a Question and Answer game. A player steps forward for a correct answer, but steps backward for an incorrect answer. During the first round, Michael takes five steps backward. Jacob takes three steps forward. Emily takes three steps backward. Austin does not move. Sarah takes two steps forward.

A. 1. Which integer describes Austin's position in the game?

2. Draw a number line. Represent each player's position on the number line.

3. Who is in last place?

4. Which players are represented by opposites?

B. 1. In the next round each player moves two steps forward. Place all five players on a new number line.

2. Are any players that were opposites before still opposites now? Why or why not?

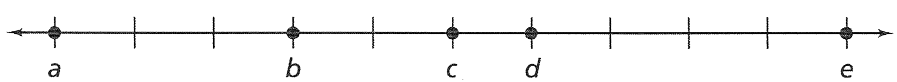
3. What does it mean when you read the numbers on the number line from the left to the right?

C. In the final round, Emily stays in the same place, and Michael is at her opposite. How many steps did Michael take in the final round?

Exercises

For Exercises 1–4, place each integer on a number line. Then identify any opposites.

1. $^{-}1, 4, 2, ^{-}4, 3, 1$
2. $2, 0, ^{-}3, 4, ^{-}1, 3$
3. $^{-}5, 10, ^{-}2, 4, 0, ^{-}10$
4. $^{-}5, 8, ^{-}7, ^{-}10, 5, 10$
5. Use an integer to represent each play in a football game.
   - **a.** The fullback carries the ball for a gain of 6 yards.
   - **b.** The quarterback is sacked for a loss of 3 yards.
   - **c.** The play stops at the line of scrimmage for no gain.
6. Use an integer to represent each change to a bank account.
   - **a.** A deposit of $20 is made on Monday.
   - **b.** A check for $4 is written on Tuesday.
   - **c.** A check for $6 is written on Wednesday.
   - **d.** No transactions are made on Thursday.
7. Use an integer to represent each position of an elevator.
   - **a.** The elevator leaves the ground floor and arrives at the 12th floor.
   - **b.** The elevator leaves the ground floor and arrives at the second basement level.
   - **c.** The elevator leaves the ground floor, arrives at the 7th floor, and then travels down 3 floors.
8. Use an integer to represent time in seconds for a space ship launch.
   - **a.** Lift off.
   - **b.** The countdown begins with 10 seconds before lift off.
   - **c.** The space ship has been in the air for one minute.
   - **d.** Why do you think a launch countdown starts at *T-minus ten seconds?*
9. Use the number line below.



   - **a.** If a and e are opposites, what integer would you use to represent c?
   - **b.** Assign integer values to each point in part (a).
   - **c.** If a and d are opposites, is c positive or negative? Explain.

Topic 2: Understanding Integers

At a Glance

PACING 1 day

Mathematical Goals

- Compare and order positive and negative integers.

Guided Instruction

Vocabulary
- whole numbers
- negative number
- opposites
- integer

Explore extending the number line. Display a number line from $^-5$ to $^+5$ that can be copied by each student. Have students supply the natural numbers of 1, 2, 3, 4, and 5. Add the zero as you mention whole numbers. Use the number line to locate the negative integers as the opposites of the natural numbers. Draw connector arrows between each pair of opposites.

- *What is the sum of opposite integers?* (zero)
- *What is another way that you could define opposites?* (Two numbers that, when added together, have a sum of zero.)

Assign students to represent each of the students of Problem 2.1. These students move forward or backward according to their role.

- *Which integer describes Austin's position in the game?* (0)

Based on this answer, have students assign integer values to each of the other students and answer Question A.

Read Question B, and have all students take two steps forward. Students are now able to order the five integers represented by the players in the game.

Read Question C. Give the students a little time to think about their answer before the student representing Michael counts aloud the steps needed to become Emily's opposite.

Suppose the winner is the first person to reach $^+10$.

- *Who has the best chance to win on the next question?* (Jacob)
- *What point value for the question is needed by that player?* (5)
- *Where would that player be located if they got the question wrong?* (0)
- *Who is the new leader?* (Sarah)

You will find additional work on integers in the grade 7 unit *Accentate the Negative.*

ACE Assignment Guide for Topic 2

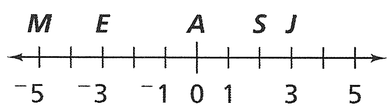
Core 1–9

Answers to Topic 2

Problem 2.1

A. 1. 0

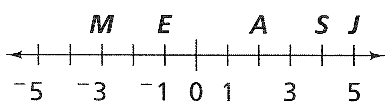
2.



3. Michael

4. Jacob and Emily

B. 1.



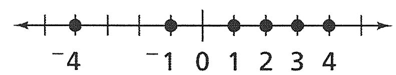
2. No, when Jacob and Emily both moved forward two steps, their relationship to the starting point (0) changed. The new positions of these players are $^{-}1$ and 5.

3. You are reading the numbers in order from the least to the greatest.

C. 4 steps; Emily started at $^{-}3$, then moved forward 2 steps to $^{-}1$. For Michael to be her opposite, Michael needs to be on $^{+}1$. Michael's last move brought him to $^{-}3$, so he needs to move forward 4 steps to be Emily's opposite.

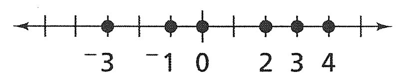
Exercises

1.



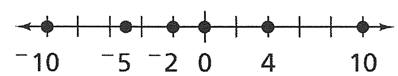
$^{-}4$ and 4

2.



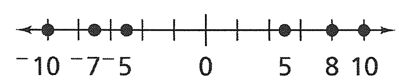
$^{-}3$ and 3

3.



$^{-}10$ and 10

4.



$^{-}10$ and 10, $^{-}5$ and 5

5. a. $^{+}6$

b. $^{-}3$

c. 0

6. a. $^{+}20$

b. $^{-}4$

c. $^{-}6$

d. 0

7. a. $^{+}12$

b. $^{-}2$

c. $^{+}7$, $^{+}4$

8. a. 0

b. $^{-}10$

c. $^{+}60$

d. The time before lift-off is negative ten seconds

9. a. 0

b. $a = {^{-}5}, c = 0, e = 5$

c. Positive; point b would be zero because it is midway between a and d. Point c is to the right of point b.

Name ______________________________ Date __________________ Period ______ Integer Unit

Comparing & Ordering Fractions & Integers Practice

(6.5.C To be taught in Integer Unit)

< is the "less than" symbol	We use this symbol to indicate that the first number listed is less than the second. Examples: • $14\frac{3}{5} < 14\frac{2}{3}$ because $14\frac{3}{5}$ is less than $14\frac{2}{3}$ • $3\frac{1}{4} < 3\frac{3}{10}$ because $3\frac{1}{4}$ is less than $3\frac{3}{10}$
> is the "greater than" symbol	We use this symbol to indicate that the first number listed is greater than the second. Examples: • $11\frac{3}{7} > 11\frac{3}{10}$ because $11\frac{3}{7}$ is greater than $11\frac{3}{10}$ • $4\frac{4}{7} > 4\frac{2}{5}$ because $4\frac{4}{7}$ is greater than $4\frac{2}{5}$

1. Insert <, >, or = in order to make each number statement accurate.

a. $\frac{7}{10}$ __________ $\frac{694}{1000}$

b. $43\frac{9}{25}$ _______ $43\frac{3}{5}$

c. $5\frac{90}{100}$ __________ $5\frac{9}{10}$

d. $17\frac{2}{25}$ __________ $17\frac{1}{5}$

e. $4\frac{1}{3}$ __________ $4\frac{3}{7}$

e. $28\frac{6}{200}$ __________ $28\frac{3}{100}$

2. Place these groups of numbers from least to greatest.

a. $\frac{11}{25}, \frac{4}{7}, \frac{3}{8}, \frac{1}{3}$

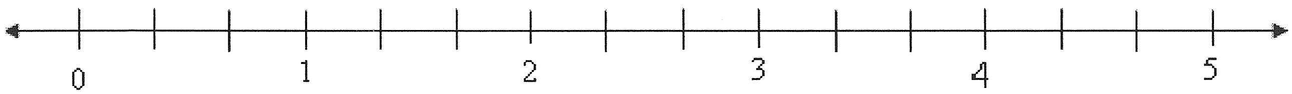
b. $5\frac{3}{7}$, $53\frac{1}{12}$, $5\frac{2}{5}$, $5\frac{1}{2}$

c. $\frac{7}{8}, \frac{9}{10}, \frac{19}{20}, \frac{2}{3}$

d. $43\frac{1}{3}, 4\frac{3}{5}, 43\frac{1}{2}, 48\frac{9}{10}$

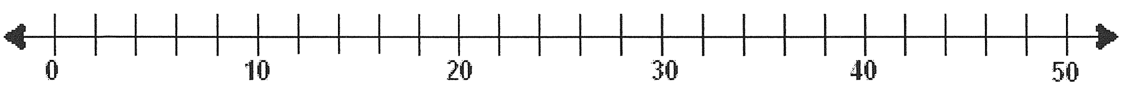
3. Place each of these numbers on the number line;

$3\frac{1}{5}, 1\frac{1}{3}, 4\frac{4}{7}, 1\frac{5}{8}, 3\frac{7}{8}$



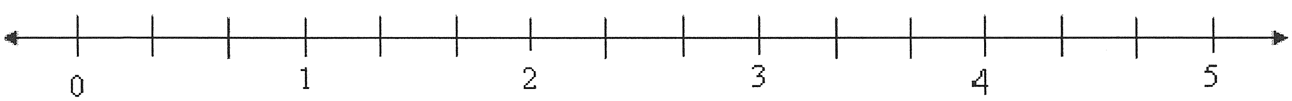
4. Place each of these numbers on the number line;

$21\frac{4}{5}, 7\frac{20}{30}, 32\frac{13}{20}, 12\frac{7}{24}, 43\frac{2}{7}$



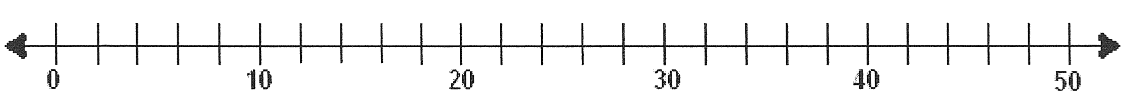
5. Place each of these numbers on the number line;

$\frac{1}{2}, 2\frac{1}{4}, 2\frac{7}{20}, 4\frac{3}{4}, 4\frac{3}{5}$



6. Place each of these numbers on the number line;

$6\frac{7}{10}, 22\frac{1}{5}, 45\frac{4}{5}, 15\frac{2}{3}, 34\frac{2}{9}$



Order of Operations Bingo

Instead of calling numbers to play Bingo, you call (and write) expressions to be evaluated for the numbers on the Bingo cards. The operations in this lesson are addition, subtraction, multiplication, and division. None of the expressions contain exponents.

Learning Objectives

Students will:

- Evaluate expressions using the order of operations on +, –, ×, and ÷
- Use mental arithmetic to evaluate expressions.

Materials

- Order of Ops Bingo Sheet
- Bowl, jar, or hat
- Chips for marking spaces on the Bingo cards

Instructional Plan

Students can often rattle off the acronym PEMDAS or "Please Excuse My Dear Aunt Sally" as being associated with the *order of operations*. Putting this memory into practice can be more of a challenge. By practicing the correct order with a motivating game of Bingo, students will be more eager to be accurate in their calculations.

Parentheses
Exponents
Multiplication / **D**ivision

Addition / **S**ubtraction

One misconception by students is that all multiplication should happen before all division because the *multiplication* comes before *division* in the acronym. In fact, multiplication and division have the same precedence and should be evaluated as they appear from left to right.

Incorrect	Correct
$12 \div 3 \times 4$	$12 \div 3 \times 4$
$12 \div 12$	4×4
1	16

Similarly, *addition* comes before *subtraction* in the acronym, yet they have the same precedence.

Incorrect	Correct
$4+10-5+8$	$4+10-5+8$
$14-13$	$14-5+8$
1	$9+8$
	17

Try giving students an additional example before starting the game.

$$2+\underline{9\div 3}-5+\underline{6\times 5}\div 2$$
$$2+3-5+\underline{30\div 2}$$
$$\underline{2+3}-5+15$$
$$\underline{5-5}+15$$
$$0+15$$
$$15$$

Playing Order of Operations Bingo

To prepare the materials for the game, you will need to print the Order of Ops Bingo Sheet. The first two pages contain 50 expression strips, which you will need to cut out and place in a bowl, jar, or hat. The third page contains two bingo cards; you will need to photocopy this sheet, cut the copies in half, and distribute a sheet to each student.

Order of Ops Bingo Sheet

The object of the game is to get five numbers in a row, vertically, horizontally, or diagonally, just as in the regular game of bingo.

> NOTE: The operations used for this lesson are addition, subtraction, multiplication, and division. None of the expressions contain exponents or parentheses.

Distribute a Bingo card to each student before starting the game. Give students the following instructions:

- Choose one space on the board as the "free" space and write the word FREE.
- Choose numbers to write into the other 24 boxes on your Bingo card. Make sure you choose numbers in the ranges given at the top of each column. That is, numbers in the first column ("B") must be in the range 1–10, numbers in

the second column ("I") must be in the range 11–20, and so on. [This ensures better distribution of the numbers.]

- You are not allowed to repeat any numbers.

Place all of the expression strips in a bowl, jar, or hat, and choose them one at a time. After each selection, write the expression on the board or overhead so students can evaluate it. Students should copy down and evaluate the expression on their own paper. For the first few turns, you may want to model how the numerical value is determined for the expression by writing in any applicable parentheses and going through the steps of evaluation. Make sure you write out the steps, just as you'd like to see the students do themselves. Once the number is determined, students can look for the number on their Bingo card and mark it with a pencil or a chip.

The value (i.e., the "answer") for each expression follows the expression on each strip, so be sure to share only the *expression*, saving the *answer* to verify a winner.

EXPRESSION	ANSWER
$2\times3+4\times5$	26

Keep picking expressions. Students should calculate the value for each expression, and then mark the square with that number on their card (if that number appears on their card, of course). When a student believes that she has correctly completed a column, row or diagonal on her card, she should yell, "Bingo!"

When the game has a potential winner, ask the student to call out the numbers that make the winning row, column, or diagonal. With the class, determine if the numbers that the winning student calls are indeed values from expressions that have been called out to check the math and verify the win.

To extend the game for another winner, change the rules to require 2 runs of 5 chips, or framing the exterior square of the board (16 pieces).

If students use chips instead of crossing off numbers with a pen or pencil, then they can exchange cards and play again. In order to start a second or subsequent game, all expressions used in the previous game are returned to the bowl, jar, or hat for a fresh start.

Questions for Students

The order of operations says to multiply and divide *first*. What does this mean?

> [It means that multiplication and division are performed before addition or subtraction. However, it does not mean that multiplication should be done before division. Multiplication and division have the same precedence, so if either multiplication or division occur with in an expression, perform these operations from left to right.]

What does it mean for addition and subtraction to have the same precedence?

> [It means that addition should not be done before subtraction. If either addition or subtraction occur with in an expression, perform these operations from left to right.]

In the expression 3 + 4 × 5 – (3 + 2), explain the order in which the operations should be performed, and evaluate the expression.

> [Operations within parentheses are done first, so add 3 + 2 = 5. This changes the expression to 3 + 4 × 5 – 5. Then, multiplication (and division, too, though there's none in this expression) are performed before addition and subtraction, so multiply 4 × 5 = 20. The expression is now reduced to 3 + 20 – 5. Finally, perform the addition and subtraction left to right to give 18.]

Assessment Options

1. Have students evaluate several expressions that contain several operation symbols.
2. Give students a list of numbers with no operation symbols, and ask them to place the symbols so that a specific result occurs.

    Example: Given the list of numbers 1 2 3 4 5, can you write in the symbols +, –, × and ÷ so that the value of the expression equals 8? Any of the symbols may be used more than once and all of the symbols don't have to be used.

    Answer: 1 + 2 × 3 – 4 + 5

3. Ask students to create some expressions of their own. In pairs or groups, students evaluate each other's expressions and see if there is agreement on the value of each expression. Note that students may agree on an incorrect value due to a misconception in the order of operations.

Extensions

1. Create and evaluate expressions that are more complex.
2. Present expressions containing exponents or nested parentheses (if students have had exposure to these concepts and their notation)
3. Ask the class to create expressions whose values are whole number from 0 through 75. This time, the columns on the Bingo cards have a range of 15: 1–15, 16–30, 31–45, 46–60, and 61–75.

Teacher Reflection

- What are the most common misconceptions students have regarding the order of operations? What can be done to break those misconceptions?
- What examples were most helpful in getting students to understand the order of operations? What other examples would help students to better understand the order of operations?

NCTM Standards and Expectations

Number & Operations 6-8

1. Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results.
2. Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.

This lesson was developed by Zoe Silver.

Order of Operations Bingo

Cut into strips, and share just the expressions with the class.

Expression	Answer
$(5\times5)\div(5\times5)$	1
$7\div7+7\div7$	2
$1+1+1\times1$	3
$2+3+4-5$	4
$3\cdot2-1$	5
$2\times4+6-8$	6
$1+2+1\times2+1\times2$	7
$2\times2+2\times2$	8
$3\times3\times3\div3$	9
$1+2+3+4$	10
$2\times3\times4-6-7$	11
$5\times4\times3\div5$	12
$((1+2)\times3)+4$	13

Expression	Answer
$2\times3+4\times5$	26
$4+5\times6-7$	27
$2\times2\times(3+4)$	28
$(3+4\cdot5)+6$	29
$9\cdot8-7\cdot6$	30
$3\times3\times3+4$	31
$4\times4+4\times4$	32
$7+6\times5-4$	33
$10\times9-8\times7$	34
$5+5+5\times5$	35
$(3+3)\times2\times3$	36
$5\times6+7$	37
$(5\cdot4\cdot4-4)\div2$	38

$1\times2+3\times4$	14
$5+4+3\times2$	15
$(1+2)\times(3+4)-5$	16
$8+8+8\div8$	17
$6\cdot5-4\cdot3$	18
$4\times5+6-7$	19
$2+((3+4)+5)+6$	20
$1+(2+3)\times4$	21
$6\times7-5\times4$	22
$6+(5\times4)-3$	23
$2\times3\times4$	24
$(1+4)\times(4+1)$	25

$3\times(4+(4+5))$	39
$(4+3+2+1)\times4$	40
$6\times7+8-9$	41
$2\times3\times(3+4)$	42
$5+6+8\times4$	43
$4\times(3\times2\times2-1)$	44
$8+7\times6-5$	45
$2\cdot(3+4\cdot5)$	46
$2\cdot3+4+5\cdot6+7$	47
$54-3\times2$	48
$7\times7\times6\div6$	49
$8+7\times6$	50

B (1 – 10)	I (11 – 20)	N (21 – 30)	G (31 – 40)	O (41 – 50)

ILLUMINATIONS NCTM marcopolo

B (1 – 10)	I (11 – 20)	N (21 – 30)	G (31 – 40)	O (41 – 50)

ILLUMINATIONS NCTM marcopolo

Investigation 4

Properties of Operations

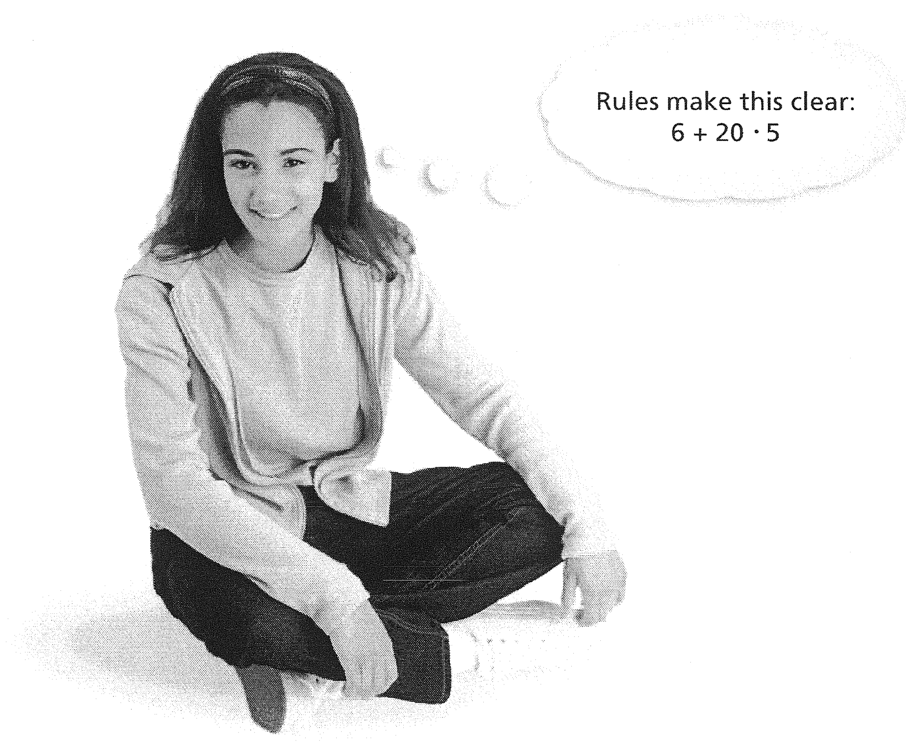
When you learn new types of numbers, you want to know what properties apply to them. You know that rational numbers are commutative for addition and multiplication.

$$-\frac{2}{3} + \frac{1}{6} = \frac{1}{6} + \left(-\frac{2}{3}\right) \text{ and } -\frac{2}{3} \times \frac{1}{6} = \frac{1}{6} \times \left(-\frac{2}{3}\right)$$

In this investigation, you will study another important property of rational numbers. You will also learn a mathematical rule that tells you the order in which to do arithmetic operations.

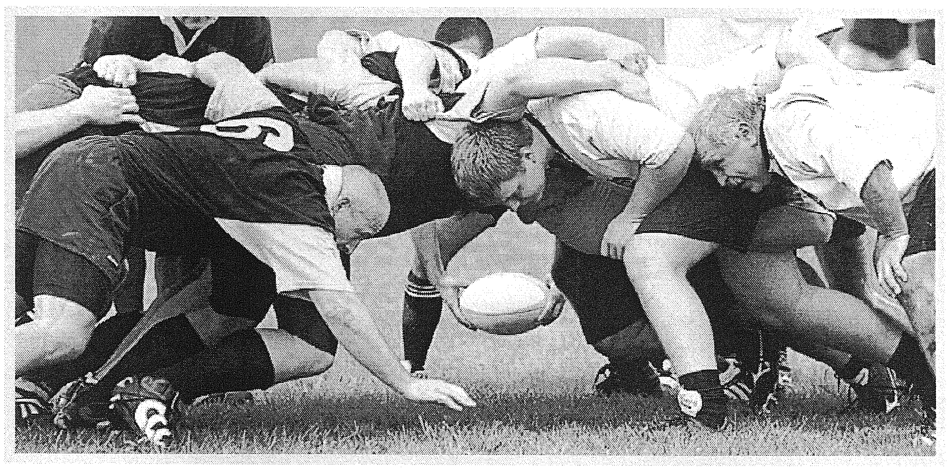
4.1 Order of Operations

Mathematicians have established rules called the **order of operations** in which to perform operations ($+$, $-$, $\times$, $\div$). Why do you need such rules?



Getting Ready for Problem 4.1

The rugby club orders 20 new jerseys. The manufacturer charges a $100 setup fee and $15 per shirt. The total cost is represented by the equation, $C = 100 + 15n$, where C is the cost in dollars and n is the number of jerseys ordered. Pedro and David calculate the amount the club owes.



Pedro's calculation:

$$\begin{aligned} C &= 100 + 15 \times 20 \\ &= 100 + 300 \\ &= \$400 \end{aligned}$$

David's calculation:

$$\begin{aligned} C &= 100 + 15 \times 20 \\ &= 115 \times 20 \\ &= \$2{,}300 \end{aligned}$$

- Who did the calculations correctly?

Order of Operations

1. Compute any expressions within parentheses.

Example 1

$(-7 - 2) + 1 =$

$-9 + 1 = -8$

Example 2

$(1 + 2) \times (-4) =$

$3 \times (-4) = -12$

2. Compute any exponents.

Example 1

$-2 + 3^2 =$

$-2 + 9 = 7$

Example 2

$6 - (-1 + 4)^2 =$

$6 - (3)^2 = -3$

3. Multiply and divide in order from left to right.

Example 1

$1 + 2 \times 4 =$ Multiplication first

$1 + 8 = 9$

Example 2

$200 \div 10 \times 2 =$ Division first

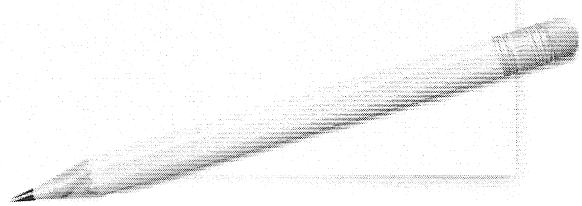
$20 \times 2 = 40$ Multiplication second

4. Add and subtract in order from left to right.

$1 + 2 - 3 \times 4 =$ Multiplication first

$1 + 2 - 12 =$ Addition and subtraction

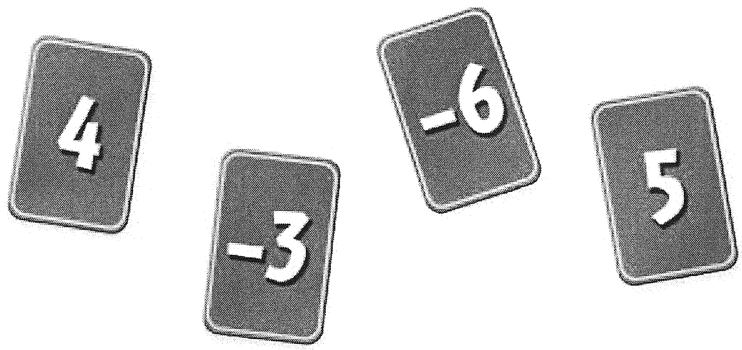
$3 - 12 = -9$



Use the order of operations in Problem 4.1.

Problem 4.1 Order of Operations

A. In a game, the goal is to write a number sentence that gives the greatest possible result using all the numbers on four cards. Jeremy draws the following four cards.



1. Joshua writes $5 - (-6) \times 4 + (-3) = 41$. Sarah says the result should be 26. Who is correct and why?
2. Wendy starts by writing $-3 - (-6) + 5^4 =$. What is her result?
3. Insert parentheses into $-3 - (-6) + 5^4$ to give a greater result than in part (2).

B. Find each value.

1. $-7 \times 4 + 8 \div 2$
2. $(3 + 2)^2 \times 6 - 1$
3. $2\frac{2}{5} \times 4\frac{1}{2} - 5^3 + 3$
4. $8 \times (4 - 5)^3 + 3$
5. $-8 \times [4 - (-5 + 3)]$
6. $-16 \div 8 \times 2^3 + (-7)$

C. Use parentheses, if needed, to make the greatest and least possible values.

1. $7 - 2 + 3^2$
2. $46 + 2.8 \times 7 - 2$
3. $25 \times (-3.12) + 21.3 \div 3$
4. $5.67 + 35.4 - 178 - 181$

D. Use the order of operations to solve this problem. Show your work.

$$3 + 4 \times 5 \div 2 \times 3 - 7^2 + 6 \div 3 = \blacksquare$$

ACE Homework starts on page 69.

4.1 Order of Operations

Goal

- Explore the use of the order of operations to order computation in problems

Launch 4.1

Use the Getting Ready to engage the class in what the challenge of the problem will be. In the Getting Ready, two students do a computation and get different answers because they did the computations in a different order. Have the students look at the problem and make their own predictions about which should be correct. They can use the context to guide the appropriate order of operations. Then turn to a discussion of the rules for the order of operations and the examples given. Return to the Getting Ready as an example and decide with the class what the answer is according to the agreed-upon rules of order. Pedro did the multiplication and then the addition, so he did the correct order of operations.

Remind students to go back to these rules as needed throughout the problem.

Talk a bit about the use of parentheses with your students. Make sure that they understand parentheses as a grouping symbol that indicates that what is in the parentheses is to be treated as a single entity. This means that you need to compute what is in the parentheses first. It also means that you can insert parentheses to make sure the expressions you write reflect the order of operations you intend. For example, for the expression $5 + (-2) \times (-3)$, you get 11 if you follow the order of operations. But suppose you want the expression to mean adding first and then multiplying. You can insert parentheses to override the order of operations. Then $(5 + (-2)) \times (-3) = -9$ because parentheses are computed first. This will be useful in Question C of Problem 4.1. Also remind students that when you need a grouping symbol like parentheses inside another parentheses, you can use brackets to make it easier to read. So, $(5 + (-2))$ becomes $[5 + (-2)]$. Since students will be working with exponents, review this notation with a few simple examples like: $3^2 = 3 \times 3, 2^3 = 2 \times 2 \times 2$ and $2^4 = 2 \times 2 \times 2 \times 2$.

Working in pairs is a good classroom arrangement for this problem.

Explore 4.1

Ask students to say in words how the mathematical sentences they write or have to interpret should be computed. For example, Question A part (1) might be read, "Find the product of −6 and 4. Subtract this product from 5. Then add −3 to the difference."

For Question C, suggest they use the order of operations rules to find an answer. Then, think about which operation can make an answer greater. Multiplication can, so you need to make the factors you multiply as great as possible. For part (2), this would suggest adding the 46 and 2.8 before you multiply. And it also suggests that you do not want to subtract the 2 from the 7 before you multiply because this decreases a factor. So $(46 + 2.8) \times 7 - 2$ should be greater. It gives a product of 339.6, whereas, without the parentheses, you get 63.6. To make it even less, you make the factors as small as possible. This suggests putting parentheses around the $7 - 2$ so that you multiply by a smaller number. With parentheses, $46 + 2.8 \times (7 - 2)$ gives 60 as the answer.

When most students have completed at least one problem in Question C, begin the whole class summary.

Summarize 4.1

Go over Question A and use the discussion to summarize the strategies students have used so far to help them both write and interpret mathematical sentences. Throughout the summary, have students say in words how the expressions they have written or been given should be computed.

Question B provides practice in using the rules for the order of operations. It also makes the point that these rules are for any numbers, including fractions.

Each of the problems in Question C asks students to find the greatest and least values. Ask students to share strategies that helped them use parentheses to make answers less and strategies that helped make answers greater.

Question D is a challenge for the students because of its length and complexity. If all students have not started this problem, give them a few minutes now to work on it before discussing it. Be sure to discuss this problem in steps so that students can reason through it and apply the order of operations when the string of symbols is long.

Have students display their thinking.

There are no parentheses, so you start with the exponent of 2 on the 7.

$3 + 4 \times 5 \div 2 \times 3 - 7^2 + 6 \div 3$
$3 + 4 \times 5 \div 2 \times 3 - 49 + 6 \div 3$

Then you continue with multiplication and division.

$3 + 4 \times 5 \div 2 \times 3 - 49 + 6 \div 3$

$3 + 20 \div 2 \times 3 - 49 + 2$

$3 + 10 \times 3 - 49 + 2$ Multiplication and division

$3 + 30 - 49 + 2$ Addition and subtraction

-14

So: $3 + 4 \times 5 \div 2 \times 3 - 7^2 + 6 \div 3 = -14$

Check for Understanding

For each example, tell the sequence of computations needed to get the correct answer and give the answer.

1. $2^2 + 7 \times (-3) - 5$
2. $(2^2 + 7) \times (-3) - 5$
3. $(2^2 + 7) \times (-3 - 5)$

Note: $(-1)^2 = 1$ but $-1^2 = -1$.
The first, $(-1)^2$, means $(-1)(-1) = 1$.
The second means $-(1)(1) = -1$.

4.1 Order of Operations

At a Glance

PACING 1 day

Mathematical Goal

- Explore the use of the order of operations to order computation in problems

Launch

Use the Getting Ready to engage the class in what the challenge of the problem will be. Have the students look at the problem and make their own predictions about which should be correct. Then turn to a discussion of the rules for the order of operations and the examples given. Return to the Getting Ready as an example and decide with the class what the answer is according to the agreed-upon rules of order.

Remind students to go back to these rules as needed throughout the problem.

Talk about the use of parentheses. Make sure that they understand parentheses as a grouping symbol that indicates that what is in the parentheses is to be treated as a single entity. You need to compute what is in the parentheses first. Also, you can insert parentheses to make sure the expressions you write reflect the order of operations you intend. Review what exponential notation means using examples like: $3^2 = 3 \times 3$, $2^3 = 2 \times 2 \times 2$ and $2^4 = 2 \times 2 \times 2 \times 2$.

Think-Pair-Share is a good classroom arrangement for this problem.

Materials

- Transparencies 4.1A, 4.1B
- Transparency markers

Vocabulary

- order of operations

Explore

Ask students to say in words how the mathematical sentences they write or have to interpret should be computed.

For Question C, suggest they use the order of operations rules to find an answer. Then think about which operation can make an answer greater.

When most students have completed at least one problem in Question C, begin the whole-class summary.

Summarize

Go over Question A and use the discussion to summarize the strategies students have used to help them both write and interpret mathematical sentences. Have students say in words how the expressions should be computed.

Question B provides practice in using the rules for the order of operations.

For Question C, ask students to share strategies that helped them use parentheses to make answers less and strategies that helped make answers greater.

Materials

- Student notebooks

continued on next page

Summarize

continued

Question D is a challenge because of its length and complexity. If all students have not started this problem, give them a few minutes now to work on it before discussing it.

Have students display their thinking and discuss the problem in steps so that students can reason through and apply the order of operations when the string of symbols is long.

Check for Understanding

For each example, tell the sequence of computations needed to get the correct answer and give the answer.

1. $2^2 + 7 \times (-3) - 5$
2. $(2^2 + 7) \times (-3) - 5$
3. $(2^2 + 7) \times (-3 - 5)$

ACE Assignment Guide for Problem 4.1

Differentiated Instruction
Solutions for All Learners

Core 2, 8–16

Other *Applications* 1, *Connections* 17–29, *Extensions* 36–43

Adapted For suggestions about ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 27: *Data About Us*, 29: *Variables and Patterns*

Answers to Problem 4.1

A. **1.** Sarah is correct. The correct sequence of computations is -6×4, which gives -24. Then subtract -24 from 5, which gives 29. Then add -3 to get 26.

2. $-3 - (-6) + 5^4 = 3 + 5^4 = 628$

3. $[-3 - (-6) + 5]^4 = 4{,}096$

B. **1.** -24 **2.** 149

3. -111.2 **4.** -5

5. -48 **6.** -23

C. The greatest and least values for each are:

1. $(7 - 2 + 3)^2 = 8^2 = 64$

$7 - (2 + 3)^2 = 7 - 25 = -18$

2. $(46 + 2.8) \times 7 - 2 = 339.6$

$46 + 2.8 \times (7 - 2) = 60$

3. $25 \times (-3.12 + 21.3) \div 3 = 151.5$

$[25 \times (-3.12)] + (21.3 \div 3) = -70.9$

4. $5.67 + 35.4 - (178 - 181) = 44.07$

$5.67 + 35.4 - 178 - 181 = -317.03$

D. $3 + 4 \times 5 \div 2 \times 3 - 7^2 + 6 \div 3 = -14$; $7^2 = 49$; $4 \times 5 = 20$; $20 \div 2 = 10$; $10 \times 3 = 30$; $6 \div 3 = 2$; $3 + 30 - 49 + 2 = -14$

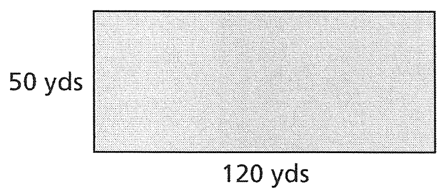
4.2 Distributing Operations

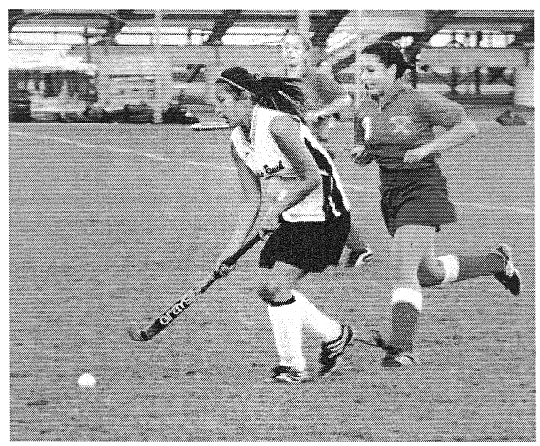
In this problem, you will compute areas of rectangles using different expressions. Look for ways to rewrite an expression into an equivalent expression that is easier to compute.

Problem 4.2 Distributing Operations

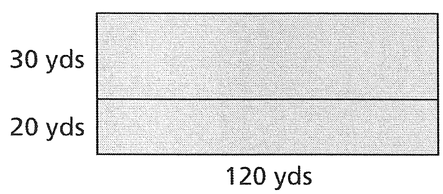
A. Richard lives in a neighborhood with a rectangular field. Each part below shows a way to divide the field for different kinds of sports.

1. Find the area.





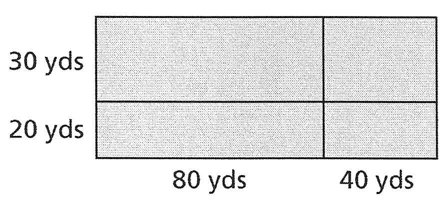
2. The field is divided into two parts.



a. Find the area of each part.

b. Write a number sentence that shows that the sum of the smaller areas is equal to the area of the entire field.

3. The field is divided into four parts.

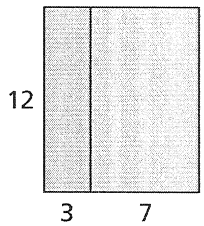


a. Find the area of each part.

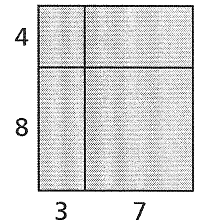
b. Write a number sentence that shows that the sum of the smaller areas is equal to the area of the entire field.

B. Use what you learned in Question A. Write two different expressions to find the area of each rectangle. Tell which uses fewer operations.

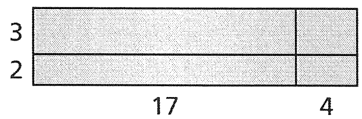
1.



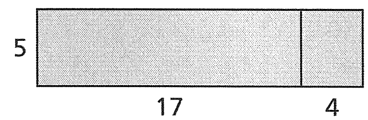
2.



3.



4.



C. **1.** Draw a rectangle whose area can be represented by $7 \times (11 + 9)$.

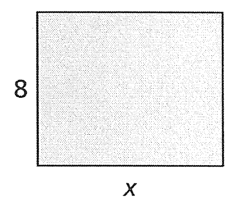
2. Write another expression for the area of the rectangle in part (1).

3. Draw a rectangle whose area can be represented by $(3 + 1) \times (3 + 4)$.

4. Write another expression for the area of the rectangle in part (3).

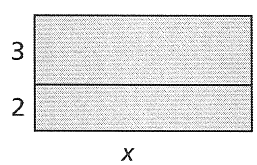
D. The unknown length in each rectangle is represented by a variable x.

1. Write an expression to represent the area of the rectangle.

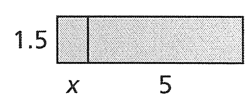


2. Write two different expressions to represent the area of each rectangle below.

a.



b.



E. Find the missing part(s) to make each sentence true.

1. $12 \times (6 + 4) = (12 \times \blacksquare) + (12 \times 4)$

2. $2 \times (n + 4) = (2 \times \blacksquare) + (\blacksquare \times 4)$

3. $(n \times 5) + (n \times 3) = \blacksquare \cdot (5 + 3)$

4. $(-3 \times 5) + (\blacksquare \times 7) = -3 \cdot (\blacksquare + 7)$

5. $4n + 11n = n \cdot (\blacksquare + \blacksquare)$

ACE **Homework starts on page 69.**

4.2 Distributing Operations

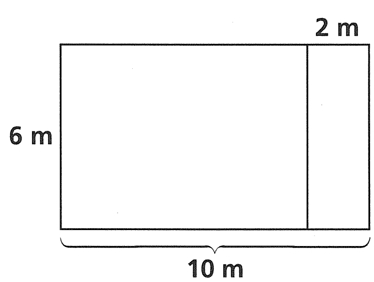
Goals

- Model the Distributive Property with areas of rectangles that have edges subdivided
- Develop and use the Distributive Property of multiplication over addition

The Distributive Property is very important to students' success in algebra. We introduce it here in a number context and will return to it in the algebra units. We have two kinds of problems to help students make sense of the Distributive Property. Contextualized problems (representing areas of rectangles) let students practice expressing the computations in the language of the situation. Number contexts let students focus on the mechanics of the Distributive Property. Even in these number situations, it is important to continue to ask students to say in words what the computations on each side of the Distributive Property mean and to suggest that students draw rectangle models if they need help in thinking about a problem.

Launch 4.2

Draw a picture of a 6 meter × 10 meter rectangle on the board. Indicate that this represents the area of a back yard where the landowner has marked off a garden across the 10-meter side that is 2 meters long.



Suggested Questions Solicit student ideas for the following questions and notate what they say.

- *What is the area of the entire backyard?* ($6\text{ m} \times 10\text{ m} = 60\text{ m}^2$)
- *What is the area of the garden?* ($6\text{ m} \times 2\text{ m} = 12\text{ m}^2$)
- *What is the area of the remaining backyard without the garden?* ($6\text{ m} \times 8\text{ m} = 48\text{ m}^2$)
- *How do these parts relate?* (whole yard = garden + rest; so $6 \times 10 = (6 \times 2) + (6 \times 8)$. Have the class check this statement.)
- *Which is easier to compute: 6×10 or $(6 \times 2) + (6 \times 8)$?* ($6 \times 10$)
- *Why would you write $(6 \times 2) + (6 \times 8)$?* (to show the area of the two sections)
- *The questions you will answer in this problem are like the problem you just analyzed. You will be trying to find expressions for the area of a rectangle and its parts that make computing or subdividing the area easy.*

If your class period is not a full hour, assign and summarize Questions A–D in class. Then assign Question E as homework to be discussed the next class day.

Have students work in small groups on this problem.

Explore 4.2

Continue to ask students to say in words what the sequence of the computations is in each of the number sentences they write.

Try to visit each group for some part of Question A or B so that you can give help, if needed.

Question C asks students to draw rectangle models for given expressions. This is asking the reverse of Questions A and B but uses what is learned in A and B.

Suggested Questions If students are struggling with Question C, ask:

- *What are the dimensions of the needed rectangle?* (7 by 20)
- *How do you show that one side is divided?* (Partition the side into 11 and 9.)

Summarize 4.2

Discuss each of the parts with your students. Look carefully at the rectangle model and make sure they understand how this model can support their thinking. You can help some students by sketching the models on the board or overhead and writing the areas on each of the sections. Some students may remember Tupelo Township from *Bits and Pieces II* in grade 6 and see that this problem is also about finding areas that make up the entire area. The difference is that here we are dealing with measures of the dimensions of parts and their areas rather than relating a fractional part to one whole.

The goal of the problem is, however, to help students to see that the Distributive Property makes sense, can make computation easier, and can help with interpretation in a context.

For Question D, if students struggle, replace the x with a box or with a number; work through the problem and then replace the box or the number with an x.

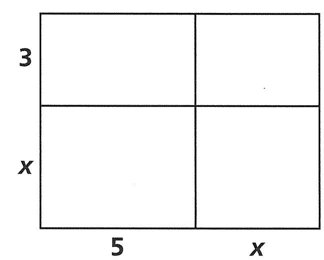
Suggested Question

- *What do these problems show us about computing the area of rectangles?* (You can compute the area of the whole rectangle. OR You can compute the areas of all the sections and add them together.)

Note that we do not take the next step to partition each side of a rectangle into two parts with x or a multiple of x as a part of each edge. This leads to multiplying binomials with variables. This is done in *Frogs, Fleas, and Painted Cubes* in grade 8. However, if your class is interested, use the following Going Further.

Going Further

- *Draw a diagram (rectangular model) to represent the following number sentences:*
  $5(3 + 6) = 5(3) + 5(6)$
  $3\frac{1}{2}(2 + 6) = 3\frac{1}{2}(2) + 3\frac{1}{2}(6)$
- *What is the total area of each rectangle?*
- *Write expressions to show two different ways to represent the area of the rectangle below.*



($(3 + x)(5 + x) = (3 \times 5) + (3 \times x) + (x \times 5) + (x \times x) = 15 + 3x + 5x + x^2 = 15 + 8x + x^2$)

4.2 Distributing Operations

At a Glance

PACING 2 days

Mathematical Goals

- Model the Distributive Property with areas of rectangles that have edges subdivided
- Develop and use the Distributive Property of multiplication over addition

Launch

Draw a picture of a 6 meter × 10 meter rectangle on the board. Indicate that this represents the area of a back yard where the landowner has marked off a garden across the 10-meter side that is 2 meters long.

- *What is the area of the entire back yard?*
- *What is the area of the garden?*
- *What is the area of the remaining yard without the garden?*
- *How do these parts relate?*
- *Which is easier to compute: 6 × 10 or (6 × 2) + (6 × 8)?*
- *Why would you write (6 × 2) + (6 × 8)?*
- *The questions you will answer in this problem are like the problem you just analyzed. You will be trying to find expressions for the area of a rectangle and its parts that make computing or subdividing the area easy.*

If your class period is not a full hour, assign and summarize Questions A–D in class. Assign Question E as homework and discuss it the next class day.

Have students work in small groups on this problem.

Materials

- Transparencies 4.2A, 4.2B

Explore

Continue to ask students to say in words what the sequence of the computations is in each of the number sentences they write.

Try to visit each group for some part of Question A or B so that you can give help, if needed.

Question C asks the reverse of Questions A and B but uses what is learned in Questions A and B. If students are struggling with Question C, ask:

- *What are the dimensions of the needed rectangle?*
- *How do you show that one side is divided?*

Summarize

Discuss each of the parts with your students. Look carefully at the rectangle model and make sure they understand how this model can support their thinking. Have students sketch the models on the board or overhead and write the areas on each of the sections.

Materials

- Student notebooks

continued on next page

Summarize

continued

For Question D, if students struggle, replace the x with a box or a number; work through the problem, then replace the box or the number with an x.

- *What do these problems show us about computing the area of rectangles?*

Going Further

- *Write expressions to show two different ways to represent the area of this rectangle. (Draw a rectangle with the dimensions: $3 + x$ by $5 + x$.)*

ACE Assignment Guide for Problem 4.2

Differentiated Instruction
Solutions for All Learners

Core 4

Other *Applications* 3, *Connections* 30–32, *Extensions* 44; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 4 and other ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 30: *Bits and Pieces I*, 32: *Variables and Patterns*

Answers to Problem 4.2

A. 1. 6,000 yd^2

2. a. The areas are 3,600 yd^2 and 2,400 yd^2.

b. $(30 \times 120) + (20 \times 120) = (20 + 30) \times 120 = 50 \times 120 = 6{,}000$ yd^2

3. a. $(30 \times 80) = 2{,}400$ yd^2;
$(20 \times 80) = 1{,}600$ yd^2;
$(30 \times 40) = 1{,}200$ yd^2;
$(20 \times 40) = 800$ yd^2;

b. $(30 \times 80) + (20 \times 80) + (30 \times 40) + (20 \times 40) = (30 + 20) \times (80 + 40) = 6{,}000$ yd^2

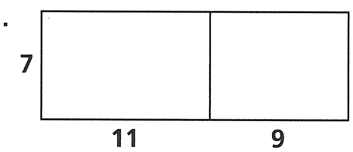
B. 1. $12 \times (3 + 7)$ or $(12 \times 3) + (12 \times 7)$; the first requires one fewer step.

2. $(4 + 8) \times (3 + 7)$ or $(4 \times 3) + (4 \times 7) + (8 \times 3) + (8 \times 7)$; the first

3. $(3 + 2) \times (17 + 4)$ or $(3 \times 17) + (3 \times 4) + (2 \times 17) + (2 \times 4)$; the first

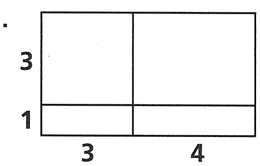
4. $5 \times (17 + 4)$ or $(5 \times 17) + (5 \times 4)$; the first

C. 1.



2. $(7 \times 11) + (7 \times 9)$

3.



4. $4 \times (3 + 4)$ or $(3 \times 3) + (3 \times 4) + (1 \times 3) + (1 \times 4)$

D. 1. $8x$

2. a. $(3 + 2) \times x$ or $3x + 2x$

b. $(5 + x) \times 1.5$ or $(5 \times 1.5) + (x \times 1.5)$

E. 1. $12 \times (6 + 4) = (12 \times \mathbf{6}) + (12 \times 4)$

2. $2 \times (n + 4) = (2 \times \mathbf{n}) + (\mathbf{2} \times 4)$

3. $(n \times 5) + (n \times 3) = \mathbf{n} \cdot (5 + 3)$

4. $(-3 \times 5) + (\mathbf{-3} \times 7) = -3 \cdot (\mathbf{5} + 7)$

5. $4n + 11n = n \cdot (\mathbf{4} + \mathbf{11})$

Name: ______________________ Date: _____________

Period: _____

Integer and Order of Operations Unit Quiz

1. Place the following numbers on the number line below:

a. -2.5

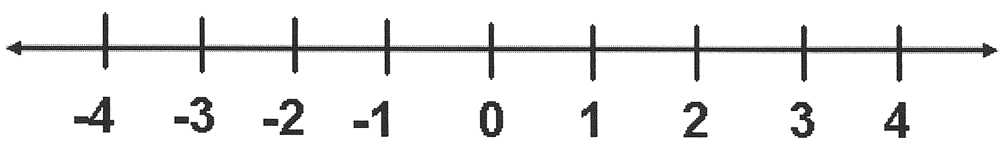
b. $\frac{2}{3}$

c. -3.25

d. 3.25

e. $\frac{11}{6}$

f. $\frac{-4}{5}$



Insert a < , >, or = to make each expression true.

2. -14 _____ -22

3. -9 ______ -3

4. $\frac{4}{10}$ ______ $\frac{2}{5}$

5. 23 ______ -23

6. -10 ______ 9

7. $\frac{-3}{4}$ ______ $\frac{-1}{4}$

8. 0 ______ -7

9. The rugby club orders 20 new jerseys. The manufacturer charges a $100 setup fee and $15 per shirt. The total cost is represented by the equation, $C = 100 + 15n$, where C is the cost in dollars and n is the number of jerseys ordered. Pedro and David calculate the amount the club owes.

Pedro's calculation:

$$\begin{aligned} C &= 100 + 15 \times 20 \\ &= 100 + 300 \\ &= \$400 \end{aligned}$$

David's calculation:

$$\begin{aligned} C &= 100 + 15 \times 20 \\ &= 115 \times 20 \\ &= \$2{,}300 \end{aligned}$$

Who did the calculations correctly? Explain your answer.

For questions 10 – 12, evaluate the expressions using the Order of Operations.

10. $3(5) + 4 \div 2$

11. $14 - 3 \times 4 + (6 - 6)$

12. $(4 + 8) \div 3 \times 5 + (2 + 9)$

Name the property illustrated in each equation.

13. (5 x 7) + (3 x 2) = (7 x 5) + (3 x 2) ____________________

14. 9 + (11 + 6) = (9 + 11) + 6 ____________________

15. 7(x + 2) = 7x + (14) ____________________

16. $\frac{3}{5} + (\frac{1}{5} + y) = (\frac{3}{5} + \frac{1}{5}) + y$ ____________________

Use mental math and number properties to simplify each expression.

17. 25 x 102

18. (51 + 13) + (9 + 7)

19. 4 x (13 x 25)

20. Draw a model to show that 8(7) = 8(5) + 8(2).

Bits and Pieces III

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?
Review Place Value and naming decimals correctly as launch to Prob. 1.1 Estimating with Decimals pg. 5	2		
Prob. 1.2 Add & Subtract Decimals pg. 7	1		
Inv. 1 Skill Sheet: adding and subtracting decimals	1	Binder / cmp2 disk	
Comparing and Ordering decimals, fractions and integers Activity	1	binder	
Quiz on + and - decimals	1	binder	
Prob. 2.1 Relating Fraction & Decimal Multiplication pg. 21 *meets 6.1.C	2		
Replacement lesson for 2.2:Decimal Multiplication: Changing the decimal problems to fraction problems	1	binder	
Prob. 2.3 Finding Decimal Products pg. 25	1		
More practice with multiplying decimals: Skill pg. 86-87	1	binder	
Multiplying decimals Quiz	1	binder	
Prob. 3.1 Deciphering Decimal Situations pg. 36 **May need more emphasis on estimating quotients for 6.1.C.*	2		
Prob. 3.2 The Great Equalizer pg. 38	1		
Prob. 3.3 Exploring Dividing Decimals pg. 40	2		
Prob. 3.4 Representing Fractions as Decimals pg. 41	1		
More practice with dividing decimals: Skill pg. 90-91	1	binder	
Multiplying and dividing whole numbers and decimals by powers of ten	2	binder	
Activity Effect of multiplying and dividing (meets 6.1.G effect)	1	binder	
Activity Converting between Fractions, Decimals, and Percents (meets 6.3.C or can be done throughout unit)	1	Several in binder	
Inv. 3 Math Reflection pg. 49	1		

6-8 Performance Expectations

6.1.A Compare and order non-negative fractions, decimals, and integers using the number line, lists, and the symbols <, >, or =. **(#'s used are from 0-50 and all rational per Test specs)**

6.1.B Represent multiplication and division of non-negative fractions and decimals using area models and the number line, and connect each representations to the related equation.

6.1.C Estimate products and quotients of fractions and decimals.

6.1.E Multiply and divide whole numbers and decimals by 1000, 100, 10, 1, 0.1, 0.01,and 0.001.

6.1.F Fluently and accurately multiply and divide non-negative decimals.

6.1.G Describe the effect of multiplying or dividing a number by one, by zero, by a number between zero and one, and by a number greater than one.

6.1.H Solve single- and multi-step word problems involving operations with fractions and decimals and verify the solutions.

6.3.C Represent percents visually and numerically, and convert between the fractional, decimal, and percent representations of a number. Look at Test and Item Specs.

6.3.D Solve single- and multi-step word problems involving ratios, rates, and percents, and verify the solutions.

Check-Up #2	1			*6.5.A Use strategies for mental computations with non-negative whole numbers, fractions, and decimals.*
Prob. 4.1 Determining Tax pg. 50 **Inv 4 Meets 6.3.D on percents*	2			
Prob. 4.2 Computing Tips pg. 52	2			
Prob. 4.3 Finding Bargains pg. 54	1			
Prob. 5.1 Finding Percent Discounts pg. 62	1			
Inv. 4 Math Reflections pg. 61	1			
Making Circles Graphs with fractions and percents (meets PE 7.4.D)	2			Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Review Concepts from Bits and Pieces III	1			
Bits and Pieces III Unit Assessment	1			
Total Instructional Days for Bits an Pieces III:	**36 days**			

All page numbers given match the student texts.

Contents in Bits and Pieces III

- Skill worksheet: Addition and Subtracting Decimals
- Math Whizz: Decimals—year 6 worksheet
- Math Whizz: Decimals—year 5 worksheet
- Comparing & Ordering Fractions, Decimals & Integers Practice worksheet
- Comparing, Ordering, Addition, Subtraction of Decimals Quiz
- Comparing, Ordering, Addition, Subtraction of Decimals Quiz Answer Key
- Decimal Multiplication—Changing the decimal problems to fractions
- Skill worksheet: Multiplying Decimals
- Multiplication of Decimal Quiz
- Multiplication of Decimal Quiz Answer Key
- Skill worksheet: Dividing Decimals
- Multiplying and Dividing Numbers by Powers of Ten worksheets
- Effective Multiplying and Dividing worksheet
- Equivalent Fractions, Decimals and Percents worksheets
- Making Circle Graphs

Name ______________________________ Date __________ Class __________

Skill: Adding and Subtracting Decimals

Investigation 1

Bits and Pieces III

First estimate. Then find each sum or difference.

1. 0.6 + 5.8

2. 2.1 + 3.4

3. 3.4 − 0.972

4. 3.1 − 2.076

5. 8.13 − 2.716

6. 5.91 + 2.38

7. 3.086 + 6.152

8. 4.7 − 1.9

9. 9.3 − 3.9

10. 5.2 − 1.86

11. 15.98 + 26.37

12. 9.27 + 15.006

13. 5.9 − 2.803

14. 15.7 − 8.923

15. 4.19 − 2.016

16. 14.75 − 6.9264

17. 5.1 + 4.83 + 9.002

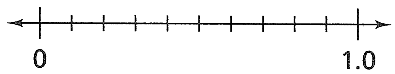
18. 3 + 4.02 + 8.6

Name ____________________ Date __________ Class __________

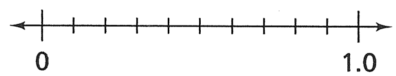
Skill: Adding and Subtracting Decimals *(continued)*

Order each set of decimals on a number line.

19. 0.2, 0.6, 0.5



20. 0.26, 0.3, 0.5, 0.59, 0.7



Use the table at the right for Exercise 21–23.

21. Find the sum of the decimals given in the chart. What is the meaning of this sum?

22. What part of the hourly work force is ages 25–44?

23. Which three age groups combined represent about one-fourth of the hourly work force?

Age of Workers Earning Hourly Pay

Age of Workers	Part of Work Force
16–19	0.08
20–24	0.15
25–34	0.29
35–44	0.24
45–54	0.14
55–64	0.08
65 & over	0.02

Decimals - Year 6

answers : ☐

Order the numbers by writing 1 (smallest) to 3 (largest) in the boxes below the numbers.

Put these numbers and measurements in order, starting with the smallest.

Q1

4.615 ☐ 4.616 ☐ 4.610 ☐

Q2

2.354 ☐ 2.345 ☐ 2.435 ☐

Q3

0.5 ☐ 0.055 ☐ 0.505 ☐

Q4

0.8 ☐ 0.088 ☐ 0.808 ☐

Q5

1.5 ☐ 1.49 ☐ 1.399 ☐

Q6

4.715 litres ☐ 4.175litres ☐ 4.517 litres ☐

Q7

2.450 kg ☐ 2.504 kg ☐ 2.405 kg ☐

Q8

1.5 kg ☐ 1.505 kg ☐ 1.05 kg ☐

Q9

3.081 litres ☐ 3.18 litres ☐ 3.8 litres ☐

Q10

2.112 kg ☐ 2.12 kg ☐ 2.2 kg ☐

Decimals - Year 5

answers : ☐

Put these numbers in order, starting with the smallest.

Q1	2.86 , 8.26 , 6.28	→	, ,
Q2	3.75 , 7.53 , 3.57 , 7.35	→	, , ,
Q3	50.01 , 10.05 , 50.10 , 10.50	→	, , ,
Q4	1.25m, 2.25m , 2.52m , 2.22m	→	m, m, m, m
Q5	9.09 , 9.90 , 0.99	→	, ,
Q6	4.10 , 5.41 , 4.51 , 5.14	→	, , ,
Q7	100.01 , 101.10 , 110.01 , 100.11	→	, , ,
Q8	£20.20 , £20.02 , £22.00 , £0.22	→	£ , £ , £ , £

Write a decimal between:

Q9	4.45	→		→	4.55
Q10	3.10	→		→	3.20
Q11	5.01	→		→	5.11
Q12	5.68	→		→	5.52
Q13	2.16	→		→	2.09
Q14	7.77	→		→	7.82
Q15	6.92	→		→	7.05

Name ___________________________ Date ________________ Period _____ Bits III

Comparing & Ordering Fractions, Decimals, & Integers Practice

(6.1.A To be taught after BP3 Prob. 1.2 Add and Subtract Decimals)

< is the "less than" symbol	We use this symbol to indicate that the first number listed is less than the second. Examples: • $14\frac{3}{5} < 14\frac{2}{3}$ because $14\frac{3}{5}$ is less than $14\frac{2}{3}$ • 3.23 < 3.3 because 3.23 is less than 3.3
> is the "greater than" symbol	We use this symbol to indicate that the first number listed is greater than the second. Examples: • $11\frac{3}{7} > 11.3$ because $11\frac{3}{7}$ is greater than 11.3 • $4.45 > 4\frac{2}{5}$ because 4.45 is greater than $4\frac{2}{5}$

1. Insert <, >, or = in order to make each number statement accurate.

a. 0.7 _______ 0.794

b. 43.36 _______ 43.6

c. 5.99 _______ $5\frac{9}{10}$

d. 17.08 _______ 17.088

e. $4\frac{1}{3}$ _______ 4.3

e. 28.03 _______ $28\frac{3}{100}$

2. Place these groups of numbers from least to greatest.

a. 0.55, 0.24, 0.245, $\frac{1}{3}$

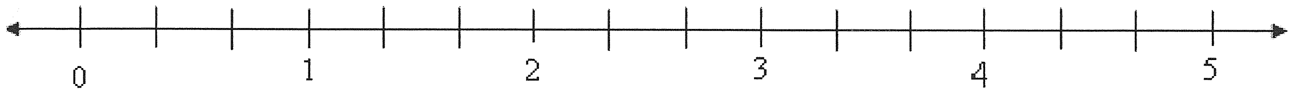
b. 5.38, 53.8, $5\frac{2}{5}$, 5.43

c. $\frac{7}{8}$, 0.22, 0.97, $\frac{2}{3}$

d. 43.33, 4.835, $43\frac{1}{2}$, $48\frac{9}{10}$

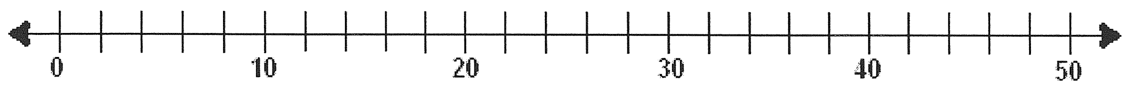
3. Place each of these numbers on the number line;

$$0.25,\ 1\frac{1}{3},\ 4.75,\ 1.25,\ 3\frac{7}{8}$$



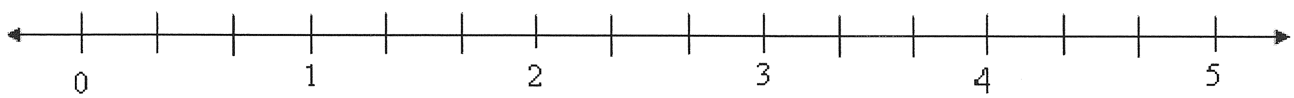
4. Place each of these numbers on the number line;

$$21.8,\ 7\frac{20}{30},\ 32.65,\ 12\frac{7}{24},\ 43\frac{2}{7}$$



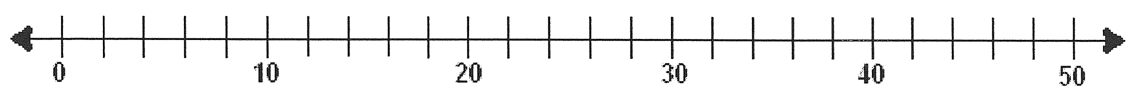
5. Place each of these numbers on the number line;

$$0.5,\ 2\frac{1}{4},\ 2.28,\ 4.65,\ 4\frac{3}{5}$$



6. Place each of these numbers on the number line;

$$6.7,\ 22\frac{1}{5},\ 45.8,\ 15.66,\ 34\frac{2}{9}$$



Comparing, Ordering, Addition, Subtraction of Decimals Quiz
Bits and Pieces III

Name____________________________ Date_____________________

1. Estimate the following and explain how you made your estimate:

   0.52 + 1.2   4.4 – 1.29

2. For each problem in question #1, find the exact sum or difference. Show your work.

3. Find the sum of the following problems. Show your work.

   1.23 + 3.9 =   4.06 + 0.357 =   3.456 + 0.9 + 14.68 =

4. Find the difference of the following problems. Show your work.

   6.00 – 4.756 =   7.45 – 5.93 =   83.64 – 49.725 =

Comparing, Ordering, Addition, Subtraction of Decimals Quiz
Bits and Pieces III

5. Insert <, >, or = in order to make each number statement accurate.

a. 0.6 ________ 0.598

b. 4.36 ________ 43.6

c. 5.89 ________ $5\frac{9}{10}$

d. 37.08 ________ 3.708

e. $4\frac{1}{3}$ ________ 4.2

e. 47.03 ________ $47\frac{3}{100}$

6. Place these groups of numbers from least to greatest.

a. 0.45, 0.2, 0.34, $\frac{1}{3}$

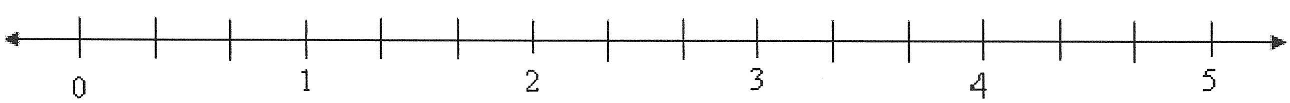
b. 4.34, 43.4, 0.434, 5.43

c. $\frac{7}{8}$, 0.45, 0.97, $\frac{2}{3}$

d. 33.33, 3.835, $33\frac{1}{2}$, $38\frac{9}{10}$

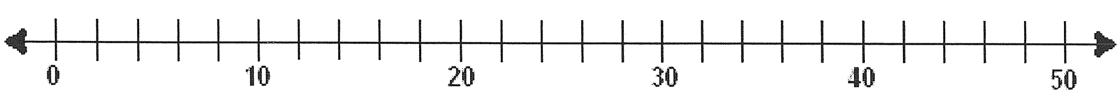
7. Place each of these numbers on the number line;

0.5, $1\frac{2}{3}$, 3.75, 2.25, $4\frac{7}{8}$



8. Place each of these numbers on the number line;

17.8, $7\frac{17}{35}$, 42.65, $2\frac{4}{21}$, $33\frac{2}{7}$



Comparing, Ordering, Addition, Subtraction of Decimals Quiz
Bits and Pieces III

Name Key Date________________

1. Estimate the following and explain how you made your estimate:

   0.52 + 1.2

   ~~0.5~~ Possible answers
   1.75
   1.50

   4.4 − 1.29

   3.25

2. For each problem in question #1, find the exact sum or difference. Show your work.

   0.52 + 1.2 = 1.72

   4.40 − 1.29 = 3.11

3. Find the sum of the following problems. Show your work.

   1.23 + 3.9 = 5.13

   4.06 + 0.357 = 4.417

   3.456 + 0.9 + 14.68 = 19.036

4. Find the difference of the following problems. Show your work.

   6.00 − 4.756 = 1.244

   7.45 − 5.93 = 1.52

   83.64 − 49.725 = 33.915

Comparing, Ordering, Addition, Subtraction of Decimals Quiz
Bits and Pieces III

5. Insert <, >, or = in order to make each number statement accurate.

a. 0.6 ___>___ 0.598 b. 4.36 ___<___ 43.6

c. 5.89 ___<___ $5\frac{9}{10}$ d. 37.08 ___>___ 3.708

e. $4\frac{1}{3}$ ___>___ 4.2 e. 47.03 ___=___ $47\frac{3}{100}$

6. Place these groups of numbers from least to greatest.

a. 0.45, 0.2, 0.34, $\frac{1}{3}$

0.2, $\frac{1}{3}$, 0.34, 0.45

b. 4.34, 43.4, 0.434, 5.43

0.434, 4.34, 5.43, 43.4

c. $\frac{7}{8}$, 0.45, 0.97, $\frac{2}{3}$

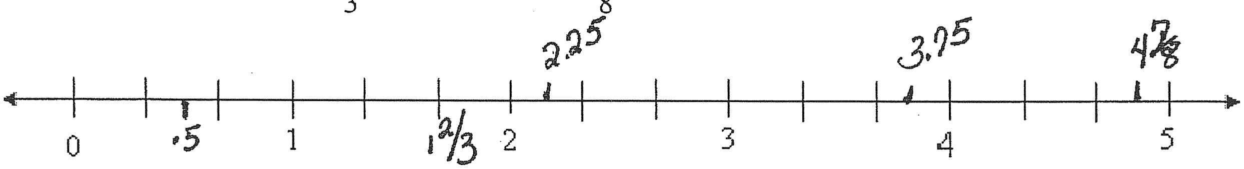
0.45, $\frac{2}{3}$, $\frac{7}{8}$, 0.97

d. 33.33, 3.835, $33\frac{1}{2}$, $38\frac{9}{10}$

3.835, 33.33, $33\frac{1}{2}$, $38\frac{9}{10}$

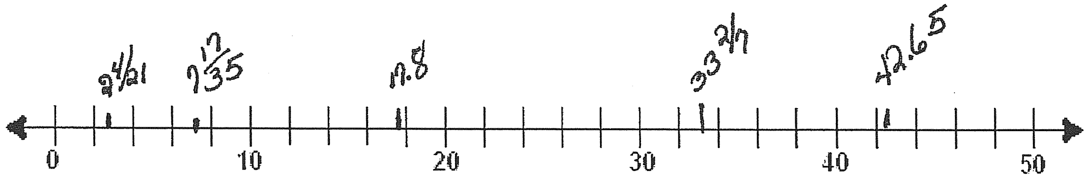
7. Place each of these numbers on the number line;

0.5, $1\frac{2}{3}$, 3.75, 2.25, $4\frac{7}{8}$



8. Place each of these numbers on the number line;

17.8, $7\frac{17}{35}$, 42.65, $2\frac{4}{21}$, $33\frac{2}{7}$



Name ______________________________ Date __________________ Period _______

Decimal Multiplication--Changing the decimal problems to fraction problems

Example: What is the answer to 125 x 5? ________

What if you changed the problem to 1.25 x 0.5? How are the products related?

To investigate the relationship, change each factor in the problem 1.25 x 0.5 to a fraction and solve.

How does changing the problem to $\frac{125}{100} \times \frac{5}{10}$ help you find the answer to 1.25 x 0.5?

Example: What number times 6 gives the product 0.36?

- First, change the problem to fractions.

  $$\frac{6}{1} \times \_\_\_\_\_ = \frac{36}{100}$$

- When we multiply fractions we multiply the numerators together and the denominators together. What fraction needs to be inserted into the problem above to make a true mathematical equation?

A. Solve the following problems by changing the decimals to fractions like the example above.

1. What number times 0.9 gives the product 2.7?

2. What number times 1.5 gives the product 0.045?

3. What number times 0.16 gives the product 0.032?

4. What number times 0.16 gives the product 0.32?

5. What number times 0.16 gives the product 3.2?

B. We know that 42 x 32 = 1344.

1. Use the factors to find two different ways to get the product 134.4.

3. Use the factors and find two numbers with a product of 0.1344.

4. Find another way to get the product 0.1344.

5. What pattern do you notice in the decimals?

C. 1. What number times 0.3 gives the product 0.9?

2. What number times 0.3 gives the product of 9?

3. What number times 0.12 gives the product 24?

Estimate each product. Then find the exact product using fraction multiplication.

0.7 x 0.6 = Estimate: Answer:	2.1 x 1.21 = Estimate: Answer:	3.82 X 5.1 = Estimate: Answer: Estimate: Answer:
0.9 x 3.1 = Estimate: Answer:	2.9 x 0.04 = Estimate: Answer:	0.62 x 4.2 = Estimate: Answer:

Name ______________________ Date __________ Class __________

Skill: Multiplying Decimals

Place the decimal point in each product.

1. $4.3 \times 2.9 = 1247$
2. $0.279 \times 53 = 14787$
3. $5.90 \times 6.3 = 3717$

Find each product.

4. 43.59×0.1
5. 246×0.01
6. 726×0.1
7. 5.342×13
8. 0.19×0.05
9. 6.4×0.09
10. 240×0.02
11. 43.79×42
12. 0.72×0.43

Skill: Multiplying Decimals *(continued)*

Use mental math to find each product.

13. 5.97×100

14. $4 \times 0.2 \times 5$

15. $3 \times (0.8 \times 1)$

16. 5.23×100

17. $0.38 \cdot 1{,}000$

18. $(5)(4.2) \times 10$

Write a number sentence you could use for each situation.

19. A pen costs \$0.59. How much would a dozen pens cost?

20. A mint costs \$0.02. How much would a roll of 10 mints cost?

21. A bottle of juice has a deposit of \$0.10 on the bottle. How much deposit money would there be on 8 bottles?

22. An orange costs \$0.09. How much would 2 dozen oranges cost?

Use $<$, $=$, or $>$ to complete each statement.

23. 2.8×10 ■ $26 \cdot 100$

24. $38.6 \cdot 10$ ■ $2 \cdot 38.6 \cdot 5$

25. 3.1×10 ■ $(0.5 \cdot 0.2)3.1$

26. $8.3 \cdot 10 \cdot 1$ ■ 8.3×100

Multiplication of Decimal Quiz (after Inv. 2 in Bits III)

Name________________________________ Date______________________________

1. Insert decimal points into the two factors, so that each of the following problems have different factors but give the same product. Explain how you made the problems.

Problem 1

201 X 15 = 30.15

Problem 2

201 X 15 = 30.15

2. Insert decimal points into the two factors, so that each of the following problems have different factors but give the same product. Explain how you made the problems.

Problem 1

111 X 25 = 27.75

Problem 2

111 X 25 = 277.5

Find the product of the following problems. Show your work.

3. 32.8 X 6.9 =

4. 4.59 X 28 =

5. 0.864 X 0.7 =

6. 79.1 X 5.8 =

7. 3.75 X 0.19 =

8. 4.56 X 6.78 =

9. 9.86 X 0.3 =

10. 75.3 X 0.06 =

Multiplication of Decimal Quiz (after Inv. 2 in Bits III)

Name **Answer key** **total points 16**

1. Insert decimal points into the two factors, so that each of the following problems have different factors but give the same product. Explain how you made the problems.

Problem 1

201 X 15 = 30.15

Possible Answers

2.01 X 15 = 30.15

201 X .15 = 30.15

Problem 2

201 X 15 = 30.15

20.1 X 1.5 = 30.15

Possible Points 4 --1 point for each correct equation & 1 point for each explanation

2. Insert decimal points into the two factors, so that each of the following problems have different factors but give the same product. Explain how you made the problems.

Problem 1

111 X 25 = 27.75

Possible Answers

1.11 X 25 = 27.75

11.1 X 2.5 = 27.75

111 X .25 = 27.75

Problem 2

111 X 25 = 277.5

11.1 X 25 = 277.5

111 X 2.5 = 277.5

Possible Points 4 --1 point for each correct equation & 1 point for each explanation

Find the product of the following problems. Show your work.

3. 32.8 X 6.9 = **226.32**

4. 4.59 X 28 = **128.52**

5. 0.864 X 0.7 **= 0.6048**

6. 79.1 X 5.8 = **458.78**

7. 3.75 X 0.19 = **0.7125**

8. 4.56 X 6.78 = **30.9168**

9. 9.86 X 0.3 = **2.958**

10. 75.3 X 0.06 = **4.518**

Possible Points 8 --- 1 point for each correct equation

Name ______________________________ Date __________ Class __________

Skill: Dividing Decimals

Investigation 3

Bits and Pieces III

Use mental math to find each quotient.

1. $7.8 \div 10$ **2.** $8.91 \div 100$ **3.** $10\overline{)46.3}$

4. $0.6 \div 10$ **5.** $1.45 \div 10$ **6.** $62.3 \div 100$

Find each quotient.

7. $0.4 \div 0.02$ **8.** $3.9 \div 0.05$ **9.** $0.2\overline{)26}$

10. $0.4\overline{)1.08}$ **11.** $0.68 \div 0.2$ **12.** $0.02\overline{)0.06}$

13. $14\overline{)889}$ **14.** $0.09\overline{)0.108}$ **15.** $0.04\overline{)0.024}$

Use <, =, or > to complete each statement.

16. $56 \div 100$ ▒ $5.6 \div 100$ **17.** $\$16.20 \div 10$ ▒ $\$162.00 \div 100$

Skill: Dividing Decimals *(continued)*

Find each quotient.

18. $1.8 \div 6$ **19.** $16\overline{)3.2}$ **20.** $17\overline{)5.1}$

21. $9\overline{)21.6}$ **22.** $15\overline{)123}$ **23.** $108 \div 5$

24. $50\overline{)17.5}$ **25.** $24\overline{)120.06}$ **26.** $9\overline{)11.24}$

Solve.

27. A package of 25 mechanical pencils costs $5.75. How much does each pencil cost?

28. A sales clerk is placing books side by side on a shelf. She has 12 copies of the same book. If the books cover 27.6 inches of the shelf, how thick is each book?

29. The salt content in the Caspian Sea is 0.13 kilograms for every liter of water. How many kilograms of salt are in 70 liters?

Find each quotient. Identify each as a terminating or repeating decimal.

30. $2.5 \div 0.08$ **31.** $9.6 \div 0.5$ **32.** $0.25 \div 0.03$

Name______________________________ Date_________________________

Record the product in each set in an organized way.

Set 1
21 x 100 =
21 x 10 =
21 x 1 =
21 x 0.1 =
21 x 0.01 =
21 x 0.001 =

Set 2
4.3 x 100 =
4.3 x 10 =
4.3 x 1 =
4.3 x 0.1 =
4.3 x 0.01 =
4.3 x 0.001 =

Set 3
0.14 x 100 =
0.14 x 10 =
0.14 x 1 =
0.14 x 0.1=
0.14 x 0.01=
0.14 x 0.001 =

1a. In Set 2, look at the product of 4.3 x 100 what happened to the decimal when multiplied by 100?

1b. Did the same thing happen in Sets 1 and 3 when we multiplied by 100?

2a. In Set 3, look at the product of 0.14 x 10 what happened to the decimal when multiplied by 10?

2b. Did the same thing happen in Sets 1 and 2?

3a. In Set 1, look at the product of 21 x 0.001 what happened to the decimal when multiplied by 0.001?

3b. Did the same thing happen in Sets 1 and 2?

4a. In Set 2, look at the product of 4.3 x 0.1 what happened to the decimal when multiplied by0.1?

5. How does multiplying by powers of ten affect the product???

Record the quotient in each set in an organized way.

Set 1
12 ÷100 =
12 ÷10 =
12 ÷1 =
12 ÷ 0.1 =
12 ÷ 0.01 =
12 ÷ 0.001 =

Set 2
3.4 ÷ 100 =
3.4 ÷ 10 =
3.4 ÷ 1 =
3.4 ÷ 0.1 =
3.4 ÷ 0.01 =
3.4 ÷ 0.001 =

Set 3
0.05 ÷ 100 =
0.05 ÷ 10 =
0.05 ÷ 1 =
0.05 ÷ 0.1=
0.05 ÷ 0.01=
0.05 ÷ 0.001 =

1a. In Set 2, look at the quotient 3.4 ÷ 100 = of what happened to the decimal when divided by 100?

1b. Did the same thing happen in Sets 1 and 3 when divided by 100?

2a. In Set 3, look at the quotient 0.05 ÷ 10 what happened to the decimal when divided by 10?

2b. Did the same thing happen in Sets 1 and 2?

3a. In Set 1, look at the quotient of 12 ÷ 0.001 what happened to the decimal when divided by 0.001?

3b. Did the same thing happen in Sets 1 and 2?

4a. In Set 2, look at the quotient of 3.4 ÷ 0.1 what happened to the decimal when divided by 0.1?

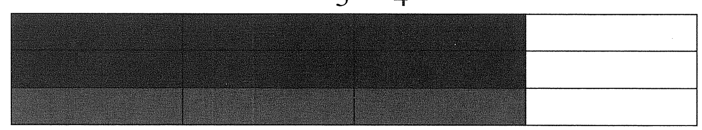
5. How does dividing by powers of ten affect the quotient???

Name______________________________ Date__________________________

1. 4.8 x 100 =	2. 0.3 x 0.1 =	3. 7.8 ÷ 10 =	4. 45 ÷ 0.01 =
5. 67 x 0.001 =	6. 9.8 ÷ 100 =	7. 64 ÷ 100 =	8. 2.5 x 0.01 =
9. 9.7 ÷ 10 =	10. 54 x 0.001 =	11. 77 ÷ 0.01 =	12. 0.63 x 0.01 =
13. 0.125 ÷ 0.1 =	14. 0.346 x 0.01=	15. 7.5 ÷ 100 =	16. 247 x 0.001 =
17. 5 x 0.001 =	18. 45 ÷ 1000 =	19. 0.789 x 1000 =	20. 85 ÷ 100 =
21. 86.5 ÷ 0.1 =	22. 78.9 x 0.001 =	23. 643 ÷ 0.1 =	24. 912 x 10 =
25. 5.4 ÷ 0.001	26. 928 ÷ 0.1=	27. 6.38 x 0.001=	28. 84.2 x 0.001=

Bits and Pieces III Effective Multiplying and Dividing (6.1.G)

Name________________________ Date____________________ Period _____

$\frac{4}{0}$ is undefined **Read the description at the right and describe what it means in your own words.**	Why can't we divide by zero? Try to divide 245÷0. Can you divide 245 into 0 groups? No! You could also ask yourself what # X zero =245. When you think about it this way, you can see that you're stuck because any number times zero is zero, not 245! Because of this situation mathematicians say that division by zero is **<u>undefined</u>**.
Multiplication and division by 1	5 X 1 = 5 Any number multiplied by 1 is the number itself. Think of having 1 group of 5 apples how many apples total? **The answer is 5 apples**. 6 ÷1 =6 Think of having 6 candy bars to be shared by 1 person how many candy bars will he have? **The answer is 6 candy bars.**
Multiplication of a fraction less than 1 by a fraction less 1 results in an answer less than 1	When you multiply two fractions that are between 0 and 1, the product is smaller than both fractions. You want to know what $\frac{2}{3} X \frac{3}{4}$ is.  Each small rectangle is $\frac{1}{12}$ of the whole and there are 6 small rectangles that are doubled colored so $\frac{2}{3} X \frac{3}{4} = \frac{6}{12}$.
Multiplication of a number greater than 1 by a fraction less than 1 but greater than zero results in an answer smaller than what you started with	3 x $\frac{3}{4}$ means $\frac{3}{4}$ of 3 which means when you solve the problem you won't have all of the 3 wholes you started with! So, 3 x $\frac{3}{4}$ = a number less than 3. Which means the answer is less than 3 but greater than 0. $\frac{3}{4} \times 3 = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\frac{1}{4}$

Bits and Pieces III Effective Multiplying and Dividing (6.1.G)

Answer each problem in the box on the right. Using both a picture and an algorithm. 1. Juanita has a pan of fudge that is $\frac{7}{8}$ full. Her brother Jorge eats $\frac{1}{2}$ of what was in the pan. How much does Jorge eat? 2. Jana has 2 whole pizzas. Rita came and ate $\frac{1}{8}$ of Jana's 2 pizzas. How much did Rita eat?	
Division of a fraction less than 1 by a fraction less than 1	You have $\frac{3}{4}$ yard of ribbon. How many $\frac{1}{8}$ - yard pieces can you cut the ribbon into? $\frac{3}{4} = \frac{6}{8}$  There are 6 -- $\frac{1}{8}$ pieces of ribbon in $\frac{3}{4}$ yard of ribbon.
Answer this problem in the box on the right. Using both a picture and an algorithm. **You have a piece of rope that is $\frac{4}{5}$ of yard long. How many $\frac{1}{10}$ --yard pieces can you cut the rope into?**	

Bits and Pieces III Effective Multiplying and Dividing (6.1.G)

Look at the problems below and predict whether the answer is going to be less than 1 or greater than and then solve the problem.

Problem	Prediction	Actual Answer
$\frac{5}{6} \text{ X } \frac{3}{4} =$		
$1\frac{7}{8} \text{ X } \frac{1}{3} =$		
$\frac{5}{8} \div \frac{1}{16} =$		
$\frac{2}{3} \div \frac{1}{6} =$		
$\frac{3}{8} \text{ X } 16 =$		
$12 \div \frac{3}{4} =$		

Name ______________________ Date _____ Period ___

Equivalent Fractions, Decimals, and Percents

Fractions	Decimals	Percents
1/4		
	0.8	
		10%
1/3		
	0.625	
		50%
2/3		
	0.75	
		16.6%
1/8		
	0.1	
		20%

NAME ______________________________ DATE _____ PERIOD ___

Equivalent Fractions, Decimals, and Percents

Fractions	Decimals	Percents
4/5		
	0.625	
		66.6%
1/6		
	0.5	
		75%
1/9		
	0.125	
		25%
1/3		
	0.2	
		10%

Name ______________________________ **Date** _____ **Period** ___

Equivalent Fractions, Decimals, and Percents

Fractions	Decimals	Percents
1/5		
	0.1	
		11.1%
2/3		
	0.8	
		33.3%
1/6		
	0.5	
		75%
5/8		
	0.125	
		25%

Name ______________________ **Date** _____ **Period** ___

Equivalent Fractions, Decimals, and Percents

Fractions	Decimals	Percents
4/5		
1/3		
2/3		
1/9		
	0.25	
	0.1	
	0.625	
	0.16	
		20%
		50%
		75%
		12.5%

Name ______________________ **Date** _____ **Period** ___

Equivalent Fractions, Decimals, and Percents

Fractions	Decimals	Percents
1/10		
	0.25	
		80%
1/2		
	0.3	
		62.5%
1/6		
	0.6	
		75%
1/5		
	0.125	
		11.1%

NAME ______________________________ **DATE** _____ **PERIOD** ___

Equivalent Fractions, Decimals, and Percents

Fractions	Decimals	Percents
4/5		
	0.1	
		33.3%
5/8		
	0.5	
		66.6%
3/4		
	0.16	
		12.5%
1/9		
	0.2	
		25%

Name ______________________________ Date ______ Period ___

Equivalent Fractions, Decimals, and Percents

Fractions	Decimals	Percents
		25%
	0.8	
1/10		
		33.3%
	0.625	
1/2		
		66.6%
	0.75	
1/6		
		12.5%
	0.1	
1/5		

NAME ____________________________ DATE _____ PERIOD ___

Equivalent Fractions, Decimals, and Percents

Fractions	Decimals	Percents
1/3		
2/3		
1/9		
1/6		
	0.8	
	0.1	
	0.2	
	0.5	
		25%
		75%
		62.5%
		12.5%

NAME ______________________________ DATE _____ PERIOD ___

Equivalent Fractions, Decimals, and Percents

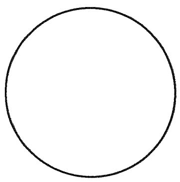
Fractions	Decimals	Percents
	0.25	
1/10		
		80%
5/8		
	0.3	
		16.6%
1/9		
		20%
	0.5	
		75%
2/3		
	0.125	

Name ______________________________ Date ______________ Period _____

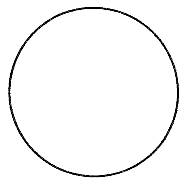
Making Circle Graphs

Shade in the fraction of each of the following circles:

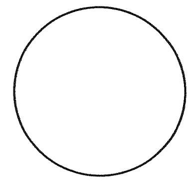
A. $\frac{1}{2}$



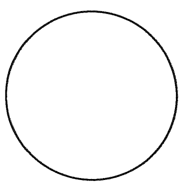
B. $\frac{1}{3}$



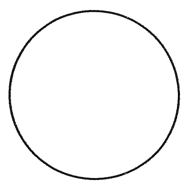
C. $\frac{1}{4}$



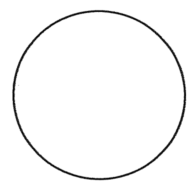
D. $\frac{1}{2}$



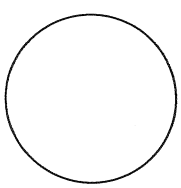
E. $\frac{1}{6}$



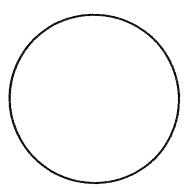
F. $\frac{1}{12}$



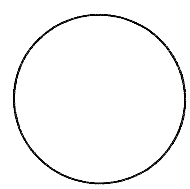
G. $\frac{1}{2}$

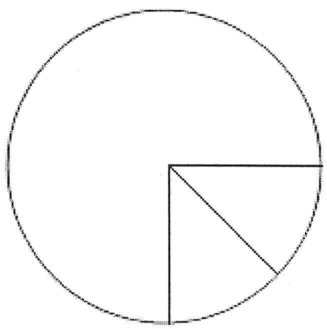


H. $\frac{1}{4}$



I. $\frac{1}{8}$





Mrs. Whoo-Ha's class was surveyed to determine their favorite ice cream flavors. The results are below.

$\frac{3}{4}$ of the class preferred cookie dough ice cream

$\frac{3}{24}$ of the class preferred mint chocolate chip ice cream

$\frac{1}{8}$ of the class preferred rocky road ice cream

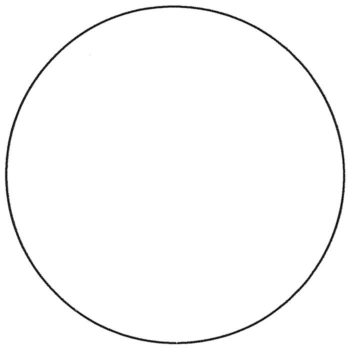
1. Change each fractional part to percents.

$\frac{3}{4}$ =

$\frac{3}{24}$ =

$\frac{1}{8}$ =

2. Label the fractional parts of the circle with the appropriate ice cream flavor.



Mrs. Schultz's class has been surveyed about their favorite pet. There are 36 students in her class.

- $\frac{12}{36}$ of the students chose dogs as their favorite pet.
- $\frac{9}{36}$ of the students chose cats as their favorite pet.
- $\frac{3}{36}$ of the students chose fish as their favorite pet.
- $\frac{6}{36}$ of the students chose turtles as their favorite pet.
- $\frac{6}{36}$ of the students chose snakes as their favorite pet.

1. Simplify each fraction. Then change each the fraction to a percent.

$\frac{12}{36} =$

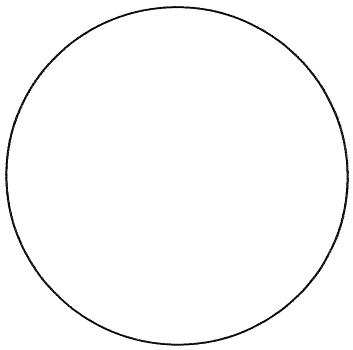
$\frac{9}{36} =$

$\frac{3}{36} =$

$\frac{6}{36} =$

$\frac{6}{36} =$

2. Represent and label the fractional parts of the circle above with the appropriate favorite pet.



Lady Gaga's stage crew was recently surveyed about their favorite fast food. There are 32 stage crew members working behind-the-scenes for Lady Gaga. Below are their results:

- $\frac{12}{32}$ of the stage crew preferred Wendy's.
- $\frac{16}{32}$ of the stage crew preferred Taco Bell.
- $\frac{4}{32}$ of the stage crew preferred KFC.

1. Simplify each fraction and then change them to percents.

$\frac{12}{32} =$

$\frac{16}{32} =$

$\frac{4}{32} =$

2. Represent and label the fractional parts of the circle with the appropriate favorite fast food.

Variables and Patterns

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations
Prob. 1.1 Preparing for a Bike Tour pg. 5	1		must do	**6.2.A Write a mathematical expression or equation with variables to represent information in a table or given situation.**
Prob. 1.2 Making Graphs pg. 7 (vocab. Important)	1		must do	
Inv. 1 Math Reflection pg. 17	1			
Prob. 2.1 Philadelphia to Atlantic City pg. 18	1		optional	**6.2.B Draw a first-quadrant graph in the coordinate plane to represent information in a table or given situation.**
Prob. 2.2 Atlantic City to Lewes pg. 20 (Vocab. important here)	1		must do	
Prob. 2.3 Lewes to Chincoteague Island pg. 22	1		optional	
Prob. 2.4 Chincoteague Island to Norfolk pg. 23	1		must do	**6.2.C Evaluate mathematical expressions when the value for each variable is given.**
Prob. 2.5 Norfolk to Williamsburg pg. 24	1		must do	
Inv. 2 Math Reflection pg. 35	1			
Check-up #1	1			**6.2.E Solve one-step equations and verify solutions.**
Prob. 3.1 Renting Bicycles pg. 37	2		optional	
Prob. 3.2 Finding Customers pg. 38	1		must do	
Prob. 3.3 Predicting Profit pg. 39	1		optional	**6.2.F Solve word problems using mathematical expressions and equations and verify solutions.**
Inv. 3 Math Reflections pg. 48	1			
***(CMP2)* Prob. 3.1 Equations with One Operation**	2	CMP2 Disk /binder		
*Evaluating Expressions given a variable (6.2.C) Activity	1	binder		Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Solving one-step Equations (Lesson AL-o) practice one step equation sheet using decimals and fractions	2	online lesson & binder		
Review for Unit Assessment	1			
Variables and Patterns Unit Assessment	1			
Total Instructional Days for Variables and Patterns:	**22 days**			

All page numbers given match the student texts.

Contents in Variables and Patterns

- CMP2 Variables and Patterns: Investigation 3.1 SE
- CMP2 Variables and Patterns: Investigation 3.1 TE
- Evaluating expressions worksheet
- Online Lesson Topic 7: Solving One-step Equations
- Practice One-step equations using rational numbers worksheet

Investigation 3

Rules and Equations

In the last investigation, you used tables and graphs of relationships to find values of one variable for given values of the other variable. In some cases, you could only estimate or predict a value.

For some relationships, you can write an equation, or formula, to show how the variables are related. Using an equation is often the most accurate way to find values of a variable.

In this investigation, you will use the patterns in tables to help you write equations for relationships. You will then use your equations to compute values of the dependent variable for specific values of the independent variable.

3.1 Writing Equations

On the last day of the Ocean Bike Tour, the riders will be near Wild World Amusement Park. Liz and Malcolm want to plan a stop there. They consider several variables that affect their costs and the time they can spend at Wild World.

Getting Ready for Problem 3.1

- What variables do you think are involved in planning for the amusement-park trip?
- How are those variables related to each other?

Malcolm finds out that it costs $21 per person to visit Wild World. Liz suggests they make a table or graph relating admission price to the number of people. However, Malcolm says there is a simple **rule** for calculating the cost:

The *cost* in dollars is equal to 21 times the *number of people.*

He writes the rule as an **equation**:

$cost = 21 \times number\ of\ people$

Liz shortens Malcolm's equation by using single letters to stand for the variables. She uses c to stand for the cost and n to stand for the number of people:

$c = 21 \times n$

When you multiply a number by a letter variable, you can leave out the multiplication sign. So, $21n$ means $21 \times n$. You can shorten the equation even more:

$c = 21n$

The equation $c = 21n$ involves one calculation. You multiply the number of customers n by the cost per customer $21. Many common equations involve one calculation.

Problem 3.1 Equations With One Operation

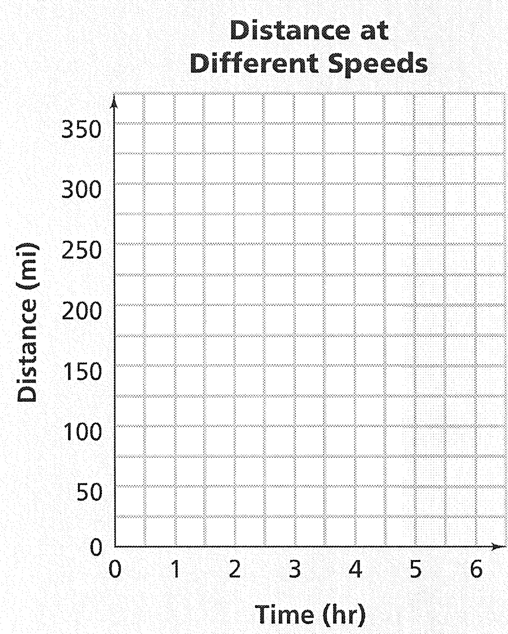
The riders visited Wild World and the tour is over. They put their bikes and gear into vans and head back to Atlantic City, 320 miles away. On their way back, they try to calculate how long the drive home will take. They use a table and a graph to estimate their travel time for different average speeds.

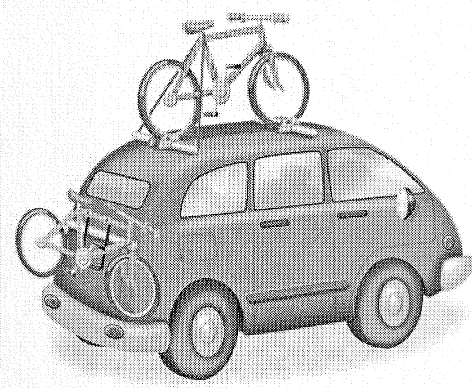
A. Copy and complete the table.

Distance Traveled at Different Average Speeds

Time (hr)	Distance for Speed of 50 mi/h	Distance for Speed of 55 mi/h	Distance for Speed of 60 mi/h
0	0	▒	▒
1	50	▒	▒
2	100	▒	▒
3	▒	▒	▒
4	▒	▒	▒
5	▒	▒	▒
6	▒	▒	▒

B. Copy and complete the graph for all three speeds below. Use a different color for each speed.





C. Do the following for each of the three average speeds:

1. Look for patterns relating distance and time in the table and graph. Write a rule in words for calculating the distance traveled in any given time.
2. Write an equation for your rule, using letters to represent the variables.
3. Describe how the pattern of change shows up in the table, graph, and equation.

D. For each speed, (50, 55, and 60 mph) tell how far you would travel in the given time. Explain how you can find each answer by using the table, the graph, and the equation.

1. 3 hours **2.** $4\frac{1}{2}$ hours **3.** $5\frac{1}{4}$ hours

E. For each speed, find how much time it will take the students to reach these cities on their route:

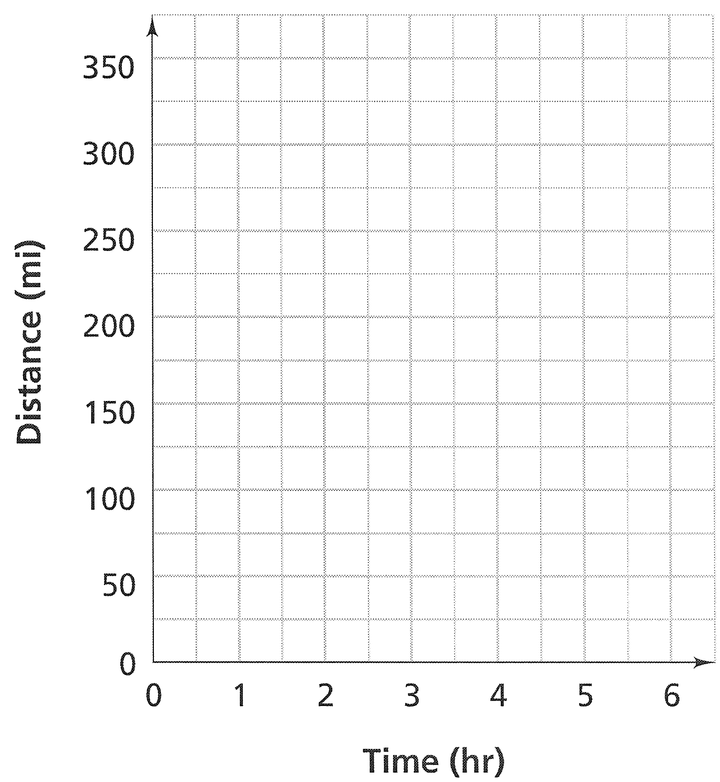
1. Atlantic City, New Jersey, about 320 miles from Norfolk
2. Baltimore, Maryland, about $\frac{3}{4}$ of the way from Norfolk to Atlantic City

ACE **Homework starts on page 55.**

Distance Traveled at Different Average Speeds

Time (hr)	Distance for Speed of 50 mi/h	Distance for Speed of 55 mi/h	Distance for Speed of 60 mi/h
0	0		
1	50		
2	100		
3			
4			
5			
6			

Distance at Different Speeds



3.1 Writing Equations

Goals

- Write one-step equations to represent relationships between variables and describe how the pattern of change shows up in a table, a graph, or an equation
- Use tables, graphs, and equations to answer questions

In this problem, students write simple one-step equations to represent relationships between variables. To help them write equations, students first write verbal rules that describe how to calculate values of one variable from values of the other. They then translate their rule to symbolic form, using letters to stand for the variables. Once they have written the equations, they use informal methods to find particular values for the dependent and independent variables. Systematic equation-solving methods are developed in later units.

Launch 3.1

Describe Liz and Malcolm's idea to plan a stop at Wild World amusement park.

Suggested Questions Discuss the Getting Ready questions.

- *What variables do you think are involved in planning for the amusement-park trip?* (Possible answers: number of people on the tour, admission costs, food costs, cost of rides, amount of time riders can spend at the park, weather)
- *How are those variables related to each other?* (Possible answers: The costs for admission, food, and rides are related to the number of people. The costs for food and rides are related to the time spent at the park. The weather is related to the amount of time spent at the park.)

Discuss the example in the student text, which shows how to write a rule for the admission costs first in words, then as an equation with words for the variables, and finally as an equation with single-letter variables. The text also discusses the convention of leaving out the multiplication sign when multiplying a number by a variable or when multiplying two variables. So, for example, $21 \times n$ may be written as $21n$. Go through the example. Ask questions to help them develop the formula as a group. Use another example if necessary.

When the students seem comfortable with the idea of writing an equation to show how variables are related, move them into small groups to work on the problem.

Explore 3.1

Some students may fill in the distance columns by using an iterative addition process, rather than by multiplying the speed by the time. For example, in the column of distances for 55 mph, they may have added 55 miles (the distance traveled in 1 hour) to each value to get the next value. These students may have difficulty coming up with a general rule linking distance and time. Ask these students how they would find the distance after 11 hours, or 16 hours, or 24 hours. They should see that extending the table to find these distances is tedious and begin to look for a more efficient method.

Suggested Questions If students are having trouble writing the rules and equations, encourage them to focus on the distances for 50 mph first. Ask:

- *What are the variables involved in this relationship?* (time, distance)
- *When you made your table, how did you find the distance for a given time?* (I multiplied 50 mph by the time.)
- *How can you state this as a rule? Start with, "The distance is equal to".* (The distance is equal to 50 times the time traveled.)
- *To write this as an equation, you need to choose letters for the variables. What letters do you want to use?* (d for distance, t for time)
- *How can you write your rule as an equation?* ($d = 50t$)

Remind students that they can refer to the example about the admission costs in their books if they need to.

Have a few groups put their table and graph on transparencies for sharing during the summary.

Summarize 3.1

Have groups share their tables and graphs.

Suggested Questions

- *In the table, how do the distance entries for the three speeds compare? Is this what you would expect?* (For each time, the faster the speed is, the greater the distance is. This makes sense because, if you drive fast for an amount of time, you go farther than if you drive slow.)
- *How do the graphs for the three speeds compare? Why does this make sense?* (For each speed, the points fall on a straight line. The faster the speed, the steeper the line. This makes sense because, for faster speeds, the distance increases faster.)

Have a few students share their rules and equations for parts (1) and (2) of Question C, and discuss the processes they used.

Suggested Questions

- *What steps did you find helpful when you wrote your rules and equations?* (Answers will vary. If students do not mention the usefulness of looking at particular examples, ask whether this was helpful and why.)
- *How did you choose letters for your variables?* (Answers will vary. Most students will use the first letter of the variable name: d for distance and t for time.) *Did you write down what each letter means so someone reading your work would understand?*

Encourage students to develop the habit of recording their decisions in statements such as, "d stands for distance measured in feet, t stands for time measured in seconds."

Discuss part (3) of Question C. Make sure the following points are made in the discussion:

- The equations are all of the same form. The only difference is the number t is multiplied by. In each case, this number is the speed of the van or the constant pattern of change seen in the table.
- The tables all show a constant pattern of change in distance values. Each time the number of hours increases by 1, the distance increases by a number equal to the speed (which is the distance traveled in 1 hour).
- The graphs all show the same pattern of change. The points are on a straight line, and faster speeds correspond to steeper lines.

NOTE: We are building a foundation for understanding the role of a constant coefficient in a simple linear equation which represents the slope of a line or the constant rate seen in the table.

Question D asks students to find distances traveled at various times. Have students present and explain their solutions. For parts (2) and (3), the easiest way to find the answers is to substitute the time values into the equations. Make sure this is discussed. For example, to find the distance traveled in $4\frac{1}{2}$ hours at 50 miles per hour is:

$d = 50 \times 4.5 = 225$

Discuss part (1) of Question E, which asks students to find the time it takes to go a given distance, 320 miles. Allow a variety of solution methods to be presented. Using the equations to answer this question is more difficult than it was for Question D. For the 50 mph case, it requires solving $320 = 50t$. Some students may use a guess-and-check approach or make estimates using the graph or the table. Other students may recognize (from the table or from practical experiences) that the distance divided by the speed equals the time, and calculate $320 \div 50$.

Part (2) adds an additional step. Students must first find $\frac{3}{4}$ of 320 miles, which is 240 miles, and then find the time required to travel that distance. Again, allow a variety of solution methods to be presented.

3.1 Writing Equations

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goals

- Write one-step equations to represent relationships between variables and describe how the pattern of change shows up in a table, a graph, or an equation
- Use tables, graphs, and equations to answer questions

Launch

Tell about Liz and Malcolm's idea to plan a stop at Wild World amusement park. Discuss the Getting Ready questions.

Discuss the example in the student text, which shows how to write a rule for the admission costs in words and as an equation. Discuss the convention of leaving out the multiplication sign when multiplying a number by a variable or when multiplying two variables.

Have students work in small groups on the problem.

Materials

- Transparency 3.1A

Vocabulary

- rule
- equation, formula

Explore

If students filled in the distance columns using iterative addition, ask them how they would find the distances after 11, 16, or 24 hours. This should help them focus on finding a more efficient method that involves multiplication.

If students are having trouble writing the rules and equations, encourage them to focus on the distances for 50 mph first.

- *What are the variables involved in this relationship?*
- *When you made your table, how did you find the distance for a given time? How can you state this as a rule? Start with, "The distance is equal to".*
- *To write this as an equation, you need to choose letters for the variables. What letters do you want to use?*
- *How can you write your rule as an equation?*

Summarize

Have students share their tables and graphs.

- *In the table, how do the distance entries for the three speeds compare? Is this what you would expect?*
- *How do the graphs for the three speeds compare? Why does this make sense?*

Have students share their rules and equations from Question C.

- *What steps did you find helpful when you wrote your rules and equations?*
- *How did you choose letters for your variables? Did you write down what each letter means so someone reading your work would understand?*

Have students present and explain their solutions for Question D. Discuss finding distance values by substituting time values into the equation.

Discuss Question E. Allow a variety of solution methods to be presented.

Materials

- Student notebooks
- Transparency 3.1B

ACE Assignment Guide for Problem 3.1

Differentiated Instruction
Solutions for All Learners

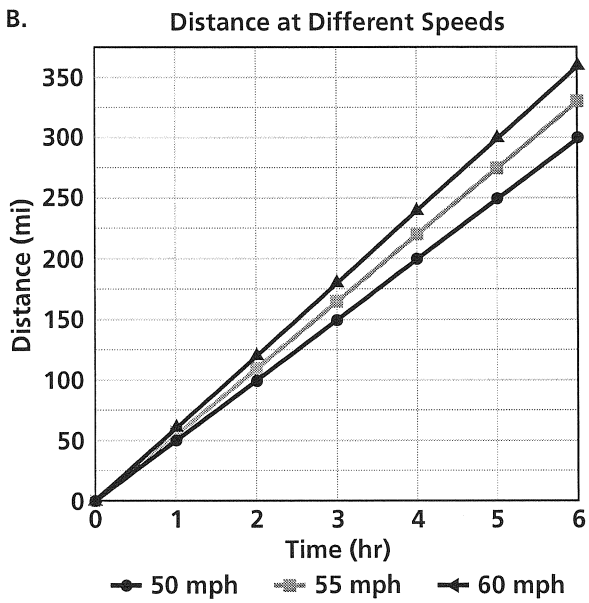
Core 1–4
Other *Connections* 21–27, *Extensions* 43

Adapted For suggestions about adapting ACE exercises, see the *CMP Special Needs Handbook.*
Connecting to Prior Units 21, 23, 26: *Covering and Surrounding*; 22: *Data About Us*; 24: *Shapes and Designs*; 25, 27: *Prime Time*

Answers to Problem 3.1

A. (Figure 1)

B.



C. 1. 50 mph: Distance equals 50 times the time.
55 mph: Distance equals 55 times the time.
60 mph: Distance equals 60 times the time.

2. $d = 50t$, $d = 55t$, $d = 60t$, where d is the distance in miles and t is the time in hours

3. For each speed, the distance increases by a constant amount each hour. In the table, this is shown by the constant difference in consecutive distance values. In the graph, this is shown by a straight-line pattern of points. In the equation, this is shown by the fact that d is equal to a number times t.

D. 1. 150 miles at 50 mph; 165 miles at 55 mph; 180 miles at 60 mph; You can read this directly from the table and estimate from the graph. For the equations, substitute 3 for t and find d.

2. 225 miles at 50 mph; 247.5 miles at 55 mph; 270 miles at 60 mph; In the table, this is the distance value halfway between the distance values for 4 hours and 5 hours. In the graph, draw a line through the set of points and find the y-coordinate of the point with x-coordinate of 4.5. To use the equations, substitute 4.5 for t and find d.

3. 262.5 miles at 50 mph; 288.75 miles at 55 mph; 315 miles at 60 mph; In the table, this is the distance $\frac{1}{4}$ of the way between the distance values for 5 hours and 6 hours. In the graph, draw a line through the set of points and find the y-coordinate of the point with a x-coordinate of 5.25. For the equations, substitute 5.25 for t and solve for d.

E. 1. $6\frac{2}{5}$ hr (or 6 hr 24 min) at 50 mph; $5\frac{9}{11}$ hr (or 5 hr 49 min) at 55 mph; $5\frac{1}{3}$ hr (or 5 hr 20 min) at 60 mph

2. $4\frac{4}{5}$ hr (or 4 hr 48 min) at 50 mph; $4\frac{4}{11}$ hr (or 4 hr 22 min) at 55 mph; 4 hr at 60 mph

Figure 1 **Distance Traveled at Different Average Speeds**

Time (hr)	Distance for Speed of 50 mi/h	Distance for Speed of 55 mi/h	Distance for Speed of 60 mi/h
0	0	0	0
1	50	55	60
2	100	110	120
3	150	165	180
4	200	220	240
5	250	275	300
6	300	330	360

6^{th} Grade Variables & Patterns Name:

Use after Investigation 3 for more advanced practice of 6.2.C evaluating expressions including decimals and fractions

Example: Evaluate 2x + 5y when x = 1.1 and y = 3.4

2 x 1.1 + 5 x 3.4

2.2 + 17.0 = 19.2

1. Evaluate 3n - 4x when n = 22.5 and x = 10.3

2. Evaluate $\frac{3}{5}x + 12$ when x = 30

3. Evaluate 2n + 1.5s when n = 31.6 and s = 18

4. Evaluate $5r - \frac{3}{4}t$ when r = 12 and $t = \frac{1}{2}$

Topic 7: Solving One-Step Equations

for use before ***Bits and Pieces II*** **Investigation 1**

The relationships between addition and subtraction or multiplication and division are called **inverse operations.** This concept of inverse operations and undoing an operation is needed to solve algebraic equations.

	Arithmetic	Algebra
Addition	$7 + 3 - 3 = 7$	$a + 3 - 3 = a$
Subtraction	$12 - 7 + 7 = 12$	$s - 7 + 7 = s$
Multiplication	$5 \times 4 \div 4 = 5$	$m \times 4 \div 4 = m$
Division	$16 \div 2 \times 2 = 16$	$d \div 2 \times 2 = d$

Problem 7.1

A. Sue knows that her plant grows 2 inches each week.

1. If g represents last week's height of the plant in inches, write an expression for the height of the plant this week.
2. Today the plant measures 16 inches in height. Set your expression equal to 16.
3. How does subtracting 2 find the height of the plant last week?
4. How tall was the plant last week?
5. What would the expression $2g$ mean?

B. Patrick just bought a book for $9. He forgot how much money he had when he entered the bookstore.

1. If m represents the amount of money he had before he bought the book, write an expression for the amount of money he has now.
2. He counts his money and finds that he has $25 left after he bought the book. Set your expression equal to 25.
3. Patrick wants to find the value of m. He does not know whether he should add or subtract 9. Determine which operation is correct and explain your decision.
4. How much money did Patrick have before he bought the book?

C. Each student pays \$4 to enter the school dance.

1. If s represents a student, write an expression for the amount of money collected for the dance.
2. The money collected totals \$168. Set your expression equal to 168. Which operation do you need to solve for s?
3. How many students came to the dance?

D. Christopher is given sheets of paper to distribute to the class for a project. He gives each student 5 sheets. He wants to know how many sheets of paper he started with.

1. Determine whether this is a multiplication or a division situation.
2. If p represents the total number of sheets of paper, write an expression for the number of students in the class.
3. There are 32 students in the class. Set your expression equal to 32.
4. How many sheets of paper did Christopher start with?

Exercises

For each Exercise 1–8, decide which operation is needed to isolate the variable. Solve the equation.

1. $a + 6 = 14$
2. $b - 3 = 9$
3. $4d = 12$
4. $7 + t = 15$
5. $\frac{x}{2} = 5$
6. $\frac{n}{9} = 6$
7. $y - 13 = 29$
8. $11h = 132$

9. Greg counted 11 people who get on the bus at the last stop. Now every seat is filled. How many people were on the bus before the stop if the bus has seats for 42 people?
10. There are four dozen daisies in a vase. If every person receives three daisies until the daisies are gone, how many people will get daisies?
11. The bulletin board has 18 square feet of space. An announcement is posted that takes up 2 square feet. How many of these announcements could be placed on this bulletin board?
12. Becky wants to solve the equation $3x = 18$. Becky says that $18 - 3 = 15$, so $x = 15$. Explain to Becky why her answer is incorrect.

Topic 7: Solving One-Step Equations

At a Glance

PACING 1 day

Mathematical Goals

- Use inverse operations to solve one-step equations.

Guided Instruction

Introduce this topic by defining and supplying several examples of inverse operations. Ask:

- *What do you get when you subtract 4 from 4?* (0)
- *What is the result of* $5 - 5$*?* (0)
- *What would you subtract from 7 to get to 0?* (7)
- *What do you get when you divide 3 by 3?* (1)
- *Simplify* $\frac{12}{12}$. (1)

Once you are satisfied that the students understand the relationships associated with identities and their inverses, begin to include variables into the discussion. Use questions like:

- *What is the result of* $b + 5 - 5$*?* (b)
- *What is the result of* $3x \div 3$*?* (x)
- *When you are trying to change an expression from* $b - 6$ *to* b*, what should you do?* (Add 6.)
- *When you are trying to change an expression from* $7b$ *to* b*, what should you do?* (Divide by 7.)

The last developmental step to the topic is to place the expressions into an equation. Use questions like these to introduce the problem.

- *What does it mean when you see an equal sign between two expressions?* (Both expressions have the same value.)
- *What happens when you add 3 to one of the expressions?* (The expressions are no longer equal.)
- *What do you need to do to keep the expressions equal?* (Add 3 to the other side as well.)
- *Why would you choose to add 3 to both sides of an equation?* (To get the variable by itself.)
- *Give an example of an equation that could be solved by adding 3 to both sides.* (Answers may vary. Sample: $t - 3 = 10$.)

You will find additional work on solving equations in the grade 7 unit *Variables and Patterns.*

Vocabulary

- inverse operations

ACE Assignment Guide for Topic 7

Core 1–12

Answers to Topic 7

Problem 7.1

A. 1. $g + 2$

2. $g + 2 = 16$

3. Answers may vary. Sample: If you know that the plant is 16 inches tall this week, and that it grew 2 inches in the past week, then subtracting 2 will allow you to work back to the height last week.

4. 14 inches

5. Answers may vary. Sample: The expression $2g$ would mean twice the height from last week.

B. 1. $m - 9$

2. $m - 9 = 25$

3. Patrick needs to add in order to leave the m by itself and solve for the value of m.

4. $34

C. 1. $4s$

2. division

3. 42 students

D. 1. Division; the whole was distributed 5 at a time. We are trying to find the whole, or total number of sheets.

2. $\frac{P}{5}$

3. $\frac{P}{5} = 32$

4. 160 sheets

Exercises

1. subtraction, $a = 8$
2. addition, $b = 12$
3. division, $d = 3$
4. subtraction, $t = 8$
5. multiplication, $x = 10$
6. multiplication, $n = 54$
7. addition, $y = 42$
8. division, $h = 12$
9. 31 people
10. 16 people
11. 9 announcements
12. Becky is trying to undo multiplication with subtraction. To solve $3x = 18$, she must divide both sides by 3. The correct solution is $x = 6$. She can check her work by substituting 6 into the original equation. $3 \times 6 = 18$.

Name:

6th grade Variables and Patterns -

Use with Success Net Solving one step equations AL-o (with fractions & decimals)

1. $x + 1.2 = 9.31$

2. $m - 3.5 = 22.6$

3. $y + \frac{3}{4} = \frac{7}{8}$

4. $12.5y = 225$

5. $\frac{w}{3} = 20$

6. $G + 2\frac{3}{8} = 5\frac{7}{8}$

7. $\frac{2}{3}x = \frac{3}{5}$

8. $\frac{P}{2.1} = 5.7$

Comparing & Scaling (CMP2)				
Investigation / Lesson / Assessments	**# of Days**	**Resource Location**	**Follow Up?**	**6-8 Performance Expectations**
Prob. 1.1 Exploring Ratios and Rates p. 6	1			**6.3.A Identify and write ratios as comparisons of part-to-part and part-to-whole relationships.** **6.3.B Write ratios to represent a variety of rates.** **6.3.D Solve single- and multi-step word problems involving ratios, rates, and percents, and verify the solutions.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Prob. 1.2 Analyzing Comparison Statements p. 7	1			
Prob. 1.3 Writing Comparison Statements p. 9	2			
Inv. 1 Math Reflections pg. 17	1			
Prob. 2.1 Developing Comparison Strategies p. 19	2			
Prob. 2.2 More Comparison Strategies p. 20-21	2			
Prob. 2.3 Scaling Ratios p. 22-23	2			
Inv. 2 Math Reflection pg. 32	1			
Check-Up #1	1	binder		
Prob. 3.1 Making and Using a Rate Table p. 34	1			
Prob. 3.2 Finding Rates pg. 35	2			
Prob. 3.3 Unit Rates and Equations *Cut part C on p. 36 and in Part E, refer to Question B (not C) when answering the questions E1 and E2	1			
Prob. 3.4 Two Different Rates p. 37-39	1			
Inv. 3 Math Reflections p. 47	1			
Comparing & Scaling Review	1			
Comparing & Scaling Unit Assessment	1			
Total Instructional Days for Comparing & Scaling:	**21 days**			

All page numbers given match the student texts.

Contents in Comparing and Scaling

- CMP2 Comparing and Scaling Check Up

Name ______________________________ Date __________ Class __________

Check-Up

for use after **Investigation 2**

A survey of 120 players in a soccer league asked which drink they preferred during and after a game.

Drink	During Game	After Game
Sports Beverage	70	10
Juice	10	80
Water	40	30

1. Write mathematical comparison statements about player drink preferences *during* the game.

a. one statement using ratios

b. one statement using percents

2. Write mathematical comparison statements about player drink preferences *after* the game.

a. one statement using fractions

b. one statement using differences

3. a. Write a ratio to describe the number of players who prefer a sports beverage to water *during* the game.

b. Write an equivalent form of the ratio.

4. The soccer league director makes the following statements based on the survey. Which statements are accurate? Explain.

   a. During the game, players prefer juice to water by a ratio of 4 to 1.

   b. 25% of the players prefer water after the game.

   c. More than half of the players prefer a sports beverage during the game.

5. So far this year, the University of North Carolina Tar Heels have won 22 games and lost 5 games.

   a. Suppose they continue at the same pace and lose 45 games. How many will they have won?

   b. What is an appropriate comparison to make between the number of games won and the number of games lost? Explain why you chose that type of comparison.

Filling and Wrapping (CMP2)

Investigation / Lesson / Assessments	# of days	Uses Teacher Express?	Follow Up?	6-8 Performance Expectations
**Be sure to use the MSP formula sheet as reference for Surface Area and Volume AFTER the understanding of how the formulas are developed*				
(CMP2) Prob. 1.1 Making Cubic Boxes pg. 6	1			**6.4.D Recognize and draw two-dimensional representations of three-dimensional figures.** **6.4.E Determine the surface area and volume of rectangular prisms using appropriate formulas and explain why the formulas work.** **6.4.F Determine the surface area of a pyramid.** **6.4.G Describe and sort polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
(CMP2) Prob. 1.2 Making Rectangular Boxes pg. 6	1			
(CMP2) Prob. 1.3 pg. 8 Rectangular Prisms	1			
(CMP2) ACE Inv. 1 p. 10-12 #1-14 Use these to help meet 6.4.D nets-drawing & recognizing (may need more practice)	1			
(CMP2) **Inv. 1 Math Reflections pg. 18**	1			
(CMP2) Prob. 2.1 Finding Surface Area pg. 20	1			
(CMP2) Prob. 2.2 Finding the Least Surface Area pg. 21	1			
(CMP2) Prob. 2.3 Finding the Volume of Rectangular Prism pg. 23	1			
More Practice with finding SA and Volume of Rectangular Prisms	1			
CMP2 Inv. 2 Math Reflections pg. 31	1			
Surface Area of Square and Triangular Pyramids	2			
Polyhedron Parts Lesson (Frank Schaefer Intro to Geo or look at the Math on Call books)	1			
Review for Filling & Wrapping	1			
Filling & Wrapping Unit Test	1			
Total Instructional Days for Filling and Wrapping:	**15 days**			

All page numbers given match the student texts.

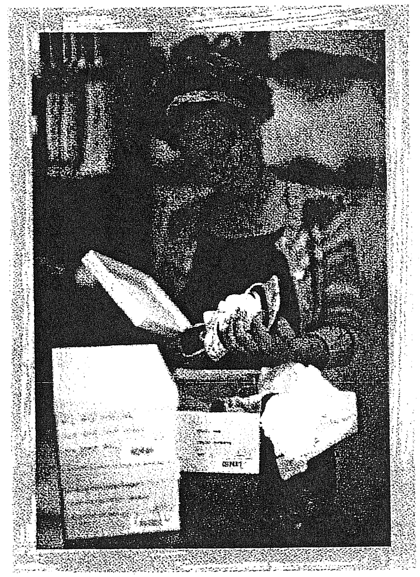
Contents in Filling and Wrapping

- CMP2 Filling and Wrapping: Investigation 1.1 SE
- CMP2 Filling and Wrapping: Investigation 1.1 TE
- CMP2 Filling and Wrapping: Investigation 1.2 SE
- CMP2 Filling and Wrapping: Investigation 1.2 TE
- CMP2 Filling and Wrapping: Investigation 1.3 SE
- CMP2 Filling and Wrapping: Investigation 1.3 TE
- CMP2 Filling and Wrapping ACE questions Investigation 1
- CMP2 Filling and Wrapping ACE questions Investigation 1 answers
- CMP2 Filling and Wrapping Mathematical Reflections Investigation 1
- CMP2 Filling and Wrapping: Investigation 2.1 SE
- CMP2 Filling and Wrapping: Investigation 2.1 TE
- CMP2 Filling and Wrapping: Investigation 2.2 SE
- CMP2 Filling and Wrapping: Investigation 2.2 TE
- CMP2 Filling and Wrapping: Investigation 2.3 SE
- CMP2 Filling and Wrapping: Investigation 2.3 TE
- CMP2 Filling and Wrapping: Mathematical Reflection Investigation 2
- Discovering Surface Area of a Pyramid worksheet
- Surface Area of Triangular and Rectangular Pyramids worksheet
- Two pages of teacher examples of different types of pyramids and total number of faces
- Polyhedron Parts Lesson

Investigation 1

Building Boxes

The most common type of package is the rectangular box. Rectangular boxes contain everything from cereal to shoes and from pizza to paper clips. Most rectangular boxes begin as flat sheets of cardboard, which are cut and then folded into a box shape.



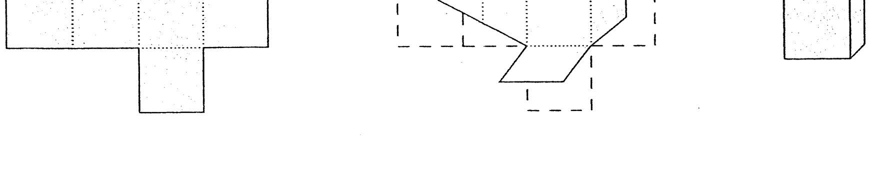
1.1 Making Cubic Boxes

Some boxes are shaped like cubes. A **cube** is a three-dimensional shape with six identical square faces.

What kinds of things might be packaged in cubic boxes?

The boxes you will work with in this problem are shaped like unit cubes. A **unit cube** is a cube with edges that are 1 unit long. For example, cubes that are 1 inch on each edge are called inch cubes. Cubes that are 1 centimeter on each edge are called centimeter cubes.

In this problem, you will make nets that can be folded to form boxes. A **net** is a two-dimensional pattern that can be folded to form a three-dimensional figure. The diagram below shows one possible net for a cubic box.



Problem 1.1 Making Cubic Boxes

On grid paper, draw nets that can be folded to make a unit cube.

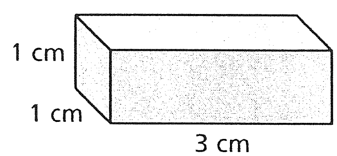
A. How many different nets can you make that will fold into a box shaped like a unit cube?

B. What is the total area of each net, in square units?

ACE Homework starts on page 10.

1.2 Making Rectangular Boxes

Many boxes are not shaped like cubes. The rectangular box below has square ends, but the remaining faces are non-square rectangles.



Problem 1.2 Making Rectangular Boxes

A. On grid paper, draw two different nets for the rectangular box above. Cut each pattern out and fold it into a box.

B. Describe the faces of the box formed from each net you made. What are the dimensions of each face?

C. Find the total area of each net you made in Question A.

D. How many centimeter cubes will fit into the box formed from each net you made? Explain your reasoning.

E. Suppose you stand the rectangular 1 centimeter × 1 centimeter × 3 centimeters box on its end. Does the area of a net for the box or the number of cubes needed to fill the box change?

active math online
For: Virtual Box Activity
Visit: PHSchool.com
Web Code: and-6102

ACE Homework starts on page 10.

1.1 Making Cubic Boxes

Goals

- Visualize a net as a representation of the surface area of a cube
- Connect the area of the net to the surface area of a cube

In this problem, students design nets on grid paper, cut them out, and fold them to form unit cubes. The area of each net is the surface area of the related cube. This problem helps students see the connection between the area of a flat figure and the surface area of a solid figure.

Students may use any size cube for this problem. However, larger cubes are easier to work with. Teachers recommend one-inch cubes and one-inch grid paper.

Launch 1.1

Discuss the work of packaging engineers, who design packages in which to store and ship objects. Packages are often designed under a set of constraints determined by the company and their customers. For example, keeping material use to a minimum is a frequently imposed constraint. Hold up some interesting boxes or containers and ask them to discuss what measurements might be useful for a packaging engineer.

Suggested Questions To help students begin thinking about packaging items, discuss the idea that some boxes are cubes. Have students describe a cube.

- *What does a cube look like?* (Examples: It is 3-dimensional; its sides look like squares, etc.)
- *What features of a cube could we count?* (its corners, its edges, its sides)
- *We call the corners* vertices. *How many vertices does a cube have?* (8)
- *How many edges does a cube have?* (12, although it may take some discussion to agree on this.)
- *We call the sides* faces. *How many faces does a cube have?* (6)

Introduce the term *unit cube*. This is a cube that is used to represent one unit of volume, or one cubic unit. A particular unit cube might be chosen as the basis unit for measuring volume, similar to the decision to measure length in inches or centimeters, or area in square inches or square centimeters. The chosen unit becomes the unit of measurement for a particular situation.

Make a copy of the net shown in the student edition, or cut it from Transparency 1.1A and display it on the overhead projector. Use it to review the special features that describe plane figures, such as dimensions, area, and perimeter. Perimeter is the distance around the net; area is the number of unit squares in the net. Corresponding measures will be developed for three-dimensional figures.

Be sure students understand that the cube cannot have 2 overlapping squares from the net. Have students work on the problem in groups of two or three to find other nets that will cover a unit cube. Each student should make at least one new net. They can cut them out to use in the summary.

Students can work in groups of 2–3 to share the work of finding all of the nets.

Explore 1.1

Suggested Questions As you circulate, ask students questions about the nets they are creating.

- *How do you know your nets will work?*
- *How could you show someone else that they will work?*
- *What things are the same in all of the nets?* (Example: They all have the same area. They all have 6 unit squares.)
- *What things are different?* (different arrangements of 6 square units of area)
- *How is the area of the net related to the number of squares that would be needed to cover the cube?*

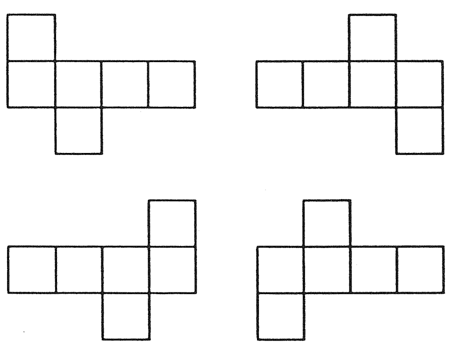
Give some groups transparent grid paper to record their nets. These can be used in the summary.

Going Further Ask students to make at least three nets for a cube without a top.

Summarize 1.1

Ask students to display the various nets on the board or overhead projector. Discuss the nets that the class generated. Repeat the questions asked in the preceding Explore section.

Students may argue that some of the nets are the same (the concept of rotational symmetry is explored in the grade 6 unit *Shapes and Designs*). Two nets are identical (congruent) if one can be flipped and turned so that it fits exactly on the other figure. For example, see the following nets. They are all congruent to one another.



Ask the following question to get students thinking about ideas that will later lead to the concept of surface area:

- *What is the total area needed to cover a unit cube?*

You may want to use ACE Exercise 32 as an in-class wrap-up problem.

At a Glance

1.1 Making Cubic Boxes

PACING 1 day

Mathematical Goals

- Visualize a net as a representation of the surface area of a cube
- Connect the area of the net to the surface area of a cube

Launch

Discuss the work of packaging engineers. To help students begin thinking about packaging items, discuss with them the introduction to the investigation in the student edition. Discuss the idea that some boxes are cubes. Have students describe a cube.

- *What does a cube look like?*
- *What features of a cube could we count? We call the corners* vertices.
- *How many vertices does a cube have? How many edges does a cube have? We call the sides* faces. *How many faces does a cube have?*

Introduce the term *unit cube*. Review the special features that describe plane figures, such as dimensions, area, and perimeter. Corresponding measures will be developed for three-dimensional figures.

Be sure students understand that the cube cannot have two overlapping squares from the net.

Each student should make at least one new net. Students can work in groups of 2–3 to share the work of finding all of the nets.

Materials
- Inch grid paper
- Inch cubes
- Transparencies 1.1A and 1.1B (optional)

Vocabulary
- Unit cube

Explore

As you circulate, ask students questions about the nets they are creating.

- *How do you know your nets will work? How could you show someone else that they will work?*
- *What things are the same in all of the nets? What things are different? How is the area of the net related to the number of squares that would be needed to cover the cube?*

Give some groups transparent grid paper to record their nets. These can be used in the summary.

Materials
- Scissors

Summarize

Ask students to display the various nets on the board or overhead projector. Discuss the nets that the class generated. Repeat the questions asked in the preceding Explore section.

- *What is the total area needed to cover a unit cube?*

You may want to use ACE Exercise 32 as an in-class wrap-up problem.

Materials
- Student notebook

ACE Assignment Guide for Problem 1.1

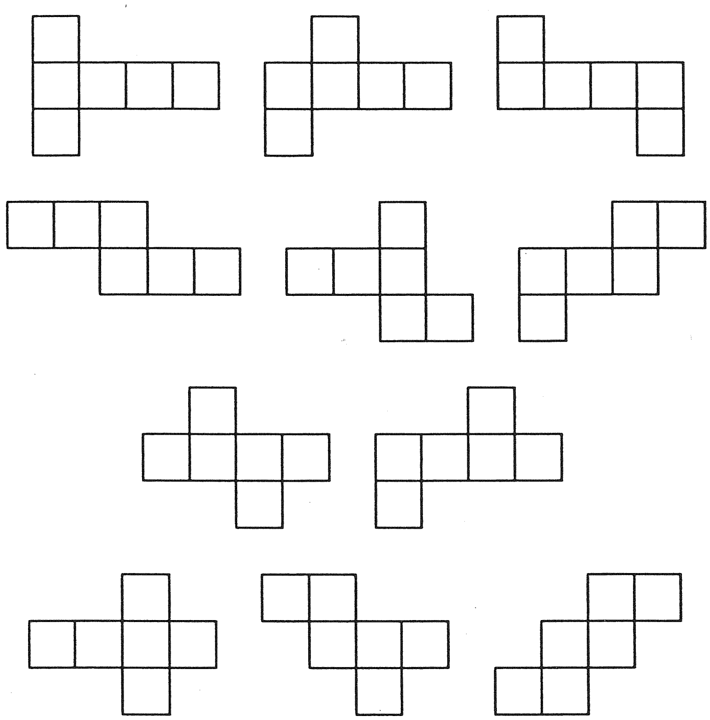
Differentiated Instruction
Solutions for All Learners

Core 1–3, 16
Other *Applications* 4, *Connections* 15, 17–18; *Extensions* 32

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 15, 18: *Covering and Surrounding*

Answers to Problem 1.1

A. There are 35 different nets that can be made with six squares (these are called hexominos). However, only the 11 shown below will fold into a cubic box. (These are shown on Transparency 1.1B.)



B. The area of each net is 6 square units. A unit cube has 6 faces, each of which has an area of 1 square unit.

Problem 1.1 Making Cubic Boxes

On grid paper, draw nets that can be folded to make a unit cube.

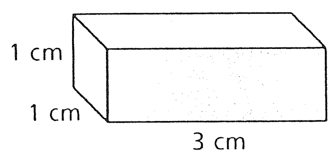
A. How many different nets can you make that will fold into a box shaped like a unit cube?

B. What is the total area of each net, in square units?

ACE Homework starts on page 10.

1.2 Making Rectangular Boxes

Many boxes are not shaped like cubes. The rectangular box below has square ends, but the remaining faces are non-square rectangles.



Problem 1.2 Making Rectangular Boxes

A. On grid paper, draw two different nets for the rectangular box above. Cut each pattern out and fold it into a box.

B. Describe the faces of the box formed from each net you made. What are the dimensions of each face?

C. Find the total area of each net you made in Question A.

D. How many centimeter cubes will fit into the box formed from each net you made? Explain your reasoning.

E. Suppose you stand the rectangular 1 centimeter × 1 centimeter × 3 centimeters box on its end. Does the area of a net for the box or the number of cubes needed to fill the box change?

active math online
For: Virtual Box Activity
Visit: PHSchool.com
Web Code: and-6102

ACE Homework starts on page 10.

1.2 Making Rectangular Boxes

Goals

- Visualize a net as a representation of the surface area of a rectangular prism
- Connect the area of the net to the surface area of a rectangular prism

In this problem, students design nets for a rectangular box.

Launch 1.2

Hold up a rectangular box that is not a cube and ask students to describe it. Discuss the features of the box—faces, edges, and vertices.

Suggested Questions Ask:

- *Describe the faces of this rectangular box.* (They are rectangles, and opposite faces are congruent.)
- *How many faces are there?* (6)
- *How many edges does the box have?* (12)
- *How many vertices does it have?* (8)
- *Will a different box have a different number of faces, edges, or vertices?*

Hold up a different box. Make sure the class realizes that, like cubes, all rectangular prisms have 6 faces, 12 edges, and 8 vertices. You may introduce the term *rectangular prism* now or wait until Problem 1.3 where it is introduced in the text.

Explain that packaging engineers may design a rectangular box by drawing a net that can be cut out and folded to make the box. Explain that the challenge is for students to find nets that will fold to form the given box.

You may want to check that students understand how to find the area of a rectangle. (If they have not worked through the *Covering and Surrounding* unit, you may want to review briefly the concepts of area and perimeter.)

Distribute centimeter grid paper. Have students investigate the problem on their own and then compare results in groups of 2 or 3.

Explore 1.2

Once students have drawn two nets and answered the questions about them, they should gather in their groups to compare their nets and validate that each could be folded into the same rectangular box.

As you circulate, continue to ask questions like those you asked in Problem 1.1. Ask students to show that their nets will work; to look for things that are the same in all of the nets; to look for things that are different; and to consider how the area of each net is related to the number of squares that would cover the rectangular box.

Summarize 1.2

Give students a chance to share their nets and show that they form the correct box. You might want to designate an area on the wall or chalkboard for the variety of nets to be displayed.

Suggested Questions Ask students how they found the area of their nets. The relationship between the area of the net and the surface area of the related box should be a focus of the discussion.

- *What was the area of your net?* (14 sq. cm)
- *How did you find that measure?* (Examples: Counting. I had one big rectangle, whose area was 12 sq. cm, and then I added on the two ends to get 14 sq. cm.)
- *What do you think the total area of the box's surface will be? Why?*
- *How do these areas compare?* (They are the same.) *Why does it make sense that these two measures are the same?*

You want to be sure to help students make the connection between the box's surface area and the area of plane figures. This is difficult for some students. You will want to revisit the idea explicitly in later problems.

In Question D, you might introduce the upcoming vocabulary of *volume*. This language is not essential at this point, but the idea is important.

- *What strategies did you use to find the number of unit cubes needed to fill the box?*

At some point in the discussion, you will want to demonstrate (or have students demonstrate) this filling. Use three cubes, with the same dimensions as the grid paper students are using, to demonstrate how they fill the rectangular boxes. If three cubes are needed to fill a box, the box has a volume of 3 unit cubes (or 3 cubic units).

Introduce the *dimensions* of a rectangular box: length, width, and height. First, the base of the box must be defined. The length and width of the base are two of the dimensions; the height of the box is the third. Make sure students are aware that placing the box on a different face changes the base: the face on the bottom will be called the base.

Check for Understanding

Give students the dimensions of a new box (preferably one you can display in front of the class) and ask them to sketch each face, labeling the dimensions and area of each face. It is best at this time to use a box with whole-number dimensions to demonstrate this.

This summary can help launch the next problem.

1.2 Making Rectangular Boxes

At a Glance

PACING 1 day

Mathematical Goals

- Visualize a net as a representation of the surface area of a rectangular prism
- Connect the area of the net to the surface area of a rectangular prism

Launch

Hold up a rectangular box that is not a cube and ask students to describe it. Discuss the features of the box.

- *Describe the faces of this rectangular box. How many faces are there?*
- *How many edges does the box have?*
- *How many vertices does it have?*
- *Will a different box have a different number of faces, edges, or vertices?*

Hold up a different box.

Explain that packaging engineers may design a rectangular box by drawing a net that can be cut out and folded to make the box. Explain the challenge for students.

Have students work on their own, then compare results in pairs or threes.

Materials

- cm grid paper
- cm cubes (optional)
- Scissors
- 2 or 3 rectangular boxes

Vocabulary

- Dimensions

Explore

Once students have drawn two nets and answered the questions about them, they should gather in their groups to compare their nets and validate that each could be folded into the same rectangular box.

Continue to ask questions like those you asked in Problem 1.1. Ask students to show that their nets will work; to look for things that are the same in all of the nets; to look for things that are different; and to consider how the area of each net is related to the number of squares that would cover the rectangular box.

Summarize

Give students a chance to share and display their nets. Ask students how they found the area of their nets. The relationship between the area of the net and the surface area of the related box should be a focus of the discussion.

- *What was the area of your net?*
- *How did you find that measure?*
- *What do you think the total area of the box's surface will be? Why?*
- *How do these areas compare?*
- *Why does it make sense that these two measures are the same?*

Materials

- Student notebook

continued on next page

Summarize

continued

Help students make the connection between the box's surface area and the area of plane figures.

- *What strategies did you use to find the number of unit cubes needed to fill the box?*

Demonstrate this filling. Use three cubes to demonstrate how they fill the rectangular boxes. Introduce the *dimensions* of a rectangular box: length, width, and height. Make sure students are aware that placing the box on a different face changes the base.

Give students the dimensions of a new box and ask them to sketch each face, labeling the dimensions and area of each face.

ACE Assignment Guide for Problem 1.2

Differentiated Instruction
Solutions for All Learners

Core 5–7

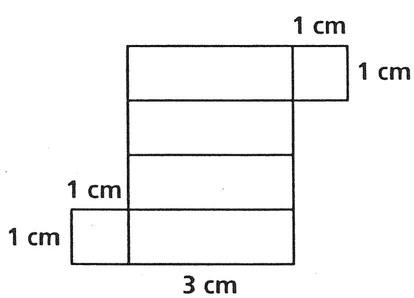
Other *Connections* 19–24; unassigned choices from previous problems

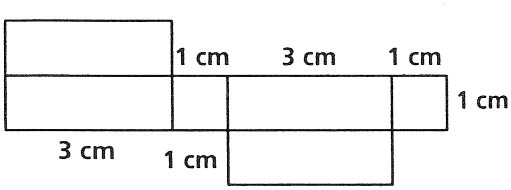
Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 19–22: *Covering and Surrounding*

Answers to Problem 1.2

A. Possible nets:





B. Four rectangular faces are congruent, with a length of 3 cm and a width of 1 cm. The remaining two faces are also congruent, with a length of 1 cm and a width of 1 cm.

C. The area for each net in Question A is 14 cm^2.

D. 3 cm cubes. One cube will cover the square end and fill one third of the box, so two more will fill the box.

E. No. If the position of the box is changed, the area of a net for the box and number of cubes needed to fill the box remain the same.

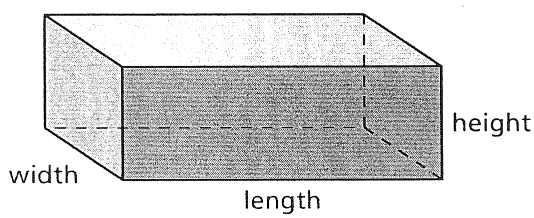
1.3 Testing Nets

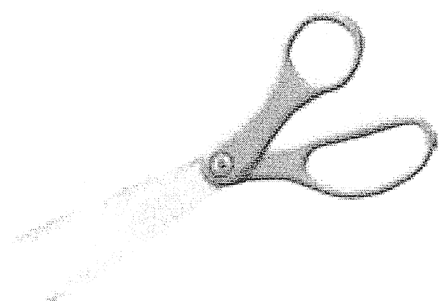
All the boxes you have made so far are rectangular prisms. A **rectangular prism** is a three-dimensional shape with six rectangular faces. The size of a rectangular prism can be described by giving its *dimensions*. The dimensions are the length, the width, and the height.

The **base** of a rectangular prism is the face on the bottom (the face that rests on the table or floor). The length and width of a prism are the length and width of its rectangular base. The height is the distance from the base of the prism to its top.

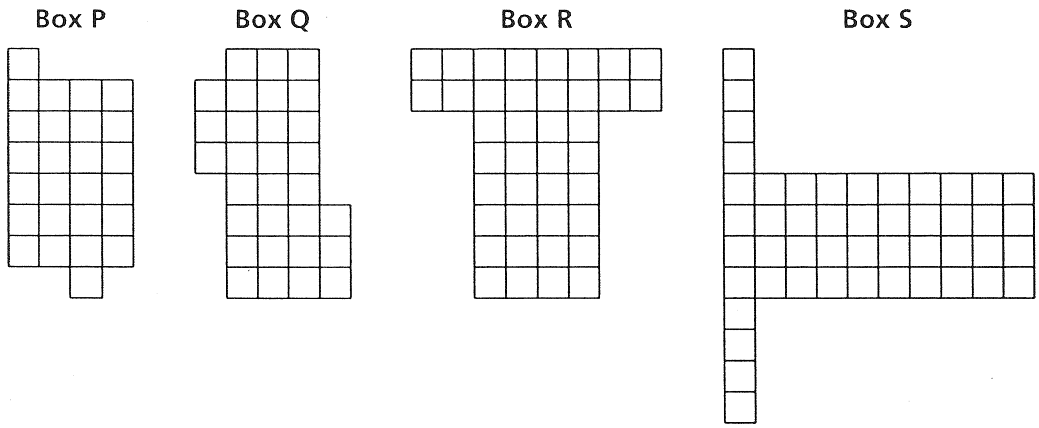
Getting Ready for Problem 1.3

- Suppose you want to cut the box in the figure below to make a net for the box. Along which edges can you make the cut?
- Are there different choices of edges to cut that will work?





An engineer at the Save-a-Tree packaging company drew the nets below. He lost the notes that indicated the dimensions of the boxes. Use your thinking from the Getting Ready section to work backwards and determine the dimensions for him.



Problem 1.3 Rectangular Prisms

A. Using a copy of the diagram above, draw in fold lines and cut each pattern and fold it to form a box. What are the dimensions of each box?

B. How are the dimensions of each box related to the dimensions of its faces?

C. What is the total area, in square units, of all the faces of each box?

D. Fill each box with unit cubes. How many unit cubes does it take to fill each box?

E. Design a net for a box that has a different shape than Box P but holds the same number of cubes as Box P.

ACE Homework starts on page 10.

1.4 Flattening a Box

Amy is a packaging engineer at the Save-a-Tree packaging company. Mr. Shu asks Amy to come to his class and explain her job to his students. She gives each student a box to do some exploring.

1.3 Testing Nets

Goals

- Visualize a net as a representation of the surface area of a rectangular prism
- Use a net for a rectangular prism to develop a strategy for finding the surface area of the prism
- Find the volume of a rectangular prism by counting the number of unit cubes it takes to fill the prism

In this problem, students make boxes from nets. The area of a net is the surface area of the related box—the amount of packaging material needed to wrap, or cover, the box. After making the boxes, students fill them with centimeter cubes to find their volumes.

Launch 1.3

Launch the problem with the Getting Ready. You might want to use an example of a rectangular prism with dimensions 2 cm by 5 cm by 8 cm.

Suggested Questions Ask:

- *What are the dimensions of each face?*
- *What patterns do you observe among the faces?*
- *What are the dimensions of the prism?*

Turn the prism to rest on another face.

- *What are the dimensions?*

To make a net out of the box; students may suggest cutting the edge labeled length, the 4 edges forming the height of the box, and the two edges in the base of the box that signify the width. There are different choices of edges to cut that would make the box fold flat to form a net.

Tell the story of the engineer who has lost his notes indicating the dimensions of each box. Distribute Labsheet 1.3 to each student.

Suggested Questions Before students begin cutting out the nets, ask:

- *Make a guess and record the dimensions of each box.* (This will help strengthen their visualization skills and understanding of dimensions.)

Students can work in groups of 2–3.

Explore 1.3

Before cutting it out, ask students to draw in the fold lines for each net. Then have them shade in one of each of the three different size faces of the box on the net. Ask how this helps find the dimensions of the box.

In Question B, if students are having difficulties finding the dimensions, have them cut out the nets and then find the dimensions and the number of unit cubes needed to fill each box. Have students fold the nets with the squares on the outside of the box so that they can check their work.

If students are having trouble finding the surface area in Question C once the net is folded, have them unfold the box to find the surface area. Students can also use an extra copy of Labsheet 1.3 for this purpose. You might want to ask some students to cut out their nets in Question E to share during the summary.

Summarize 1.3

Suggested Questions Ask students:

- *Explain how you decided where to fold each net.* (Some will have used the symmetry of the two pieces that "stick out" as the place to begin folding.)
- *What are the dimensions of each box?*
- *How can you find the dimensions from the nets? From the box? How are they related?* (Stress the importance of the base, its dimensions, and the height—the distance from the base to the top of the box.)
- *What features of the box do you observe that might make it easier to find the surface area?* (Emphasize that the faces of a box come in pairs—this will be an important idea when students develop strategies for finding surface area.)

When discussing the number of unit cubes needed to fill each box, do not go for rules—it is the filling idea that is important at this stage of students' development of the concept of volume. Some students may have already found effective ways to count the cubes—for example, by multiplying the number of cubes needed to fill the bottom of the box by the number of layers of cubes needed to fill the entire box. You could put these on the blackboard as a conjecture and come back to it. The rule or formula for finding surface area and volume will be developed in the next investigation.

Have students share their solutions to Question C. As each student displays his or her net and tells the class its dimensions and its area, ask the class whether they agree that the net works.

For Question E, put up several different nets. You may want to record the dimensions of each net, surface area, and volume in a table on poster paper.

Suggested Questions This will be useful for Investigation 2. Ask:

- *How does your new net, its dimensions, and its area compare to those for Box P?*
- *Do any of these nets have non-square faces? If not, draw a net that would have all non-square faces. (for example: $4 \times 0.5 \times 3$)*

1.3 Testing Nets

At a Glance

PACING 1 day

Mathematical Goals

- Visualize a net as a representation of the surface area of a rectangular prism
- Use a net for a rectangular prism to develop a strategy for finding the surface area of the prism
- Find the volume of a rectangular prism by counting the number of unit cubes it takes to fill the prism

Launch

Tell the story of the engineer who has lost his notes indicating the dimensions of each box. Distribute Labsheet 1.3 to each student. Before students begin cutting out the nets, ask them to guess and record the dimensions of each box. This will help strengthen their visualization skills. Students can work in groups of 2–3.

Materials
- Transparencies 1.3A and 1.3B
- Labsheet 1.3
- Scissors
- cm cubes

Vocabulary
- rectangular prism
- base

Explore

Have students cut out the nets and then find the dimensions and the number of unit cubes needed to fill each box. Have students fold the nets so the squares are on the outside of the box.

You might want to ask some students to cut out their nets in Question E to share during the summary.

Summarize

Ask students to explain how they decided where to fold each net. Some will have used the symmetry of the two pieces that "stick out" as the place to begin folding.

Discuss the dimensions of each box. Emphasize that the faces of a box come in pairs—this will be an important idea when students develop strategies for finding surface area. Stress the importance of the base, its dimensions, and the height (the distance from the base to the top of the box).

When discussing the number of unit cubes needed to fill each box, do not go for rules. The rule or formula for finding surface area and volume will be developed in the next investigation.

Have students share their solutions to Question C. As each student displays his or her net and tells the class its dimensions and its area, ask the class whether they agree that the net works. Also ask how the net, its dimensions, and its area compare to those for Box P.

Materials
- Student notebooks

ACE Assignment Guide for Problem 1.3

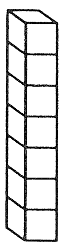
Differentiated Instruction
Solutions for All Learners

Core 8–9
Other *Connections* 25–27, 31

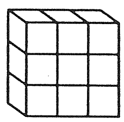
Adapted For suggestions about adapting Exercise 7 and other ACE exercises, see the CMP *Special Needs Handbook.*
Connecting to Prior Units 25, 26: *Covering and Surrounding*; 27: *Bits and Pieces II*; 31: *Bits and Pieces III*

Answers to Problem 1.3

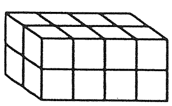
A. Box P



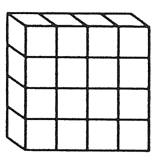
Box Q



Box R



Box S



Box P: 1 cm by 1 cm by 6 cm;
Box Q: 1 cm by 3 cm by 3 cm;
Box R: 2 cm by 2 cm by 4 cm;
Box S: 1 cm by 4 cm by 4 cm.

B. Each combination of two dimensions will yield the dimensions for a pair of congruent faces.

C. Box P: 26 cm^2; Box Q: 30 cm^2; Box R: 40 cm^2; Box S: 48 cm^2.

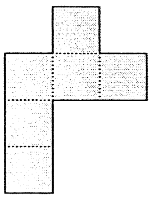
D. Box P: 6 unit cubes; Box Q: 9 unit cubes; Box R: 16 unit cubes; Box S: 16 unit cubes.

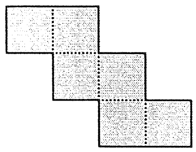
E. Answers will vary. The box should hold 6 unit cubes and have dimensions 1 cm by 2 cm by 3 cm.

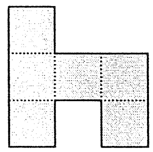
Applications Connections Extensions ACE

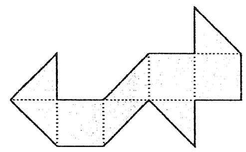
Applications

For Exercises 1–4, decide if you can fold the net along the lines to form a closed cubic box. If you are unsure, draw the pattern on grid paper and cut it out to experiment.

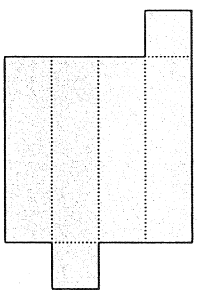
1. 

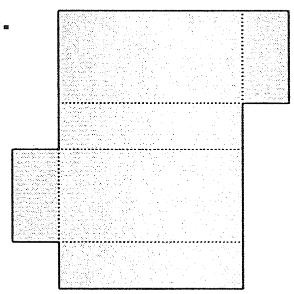
2. 

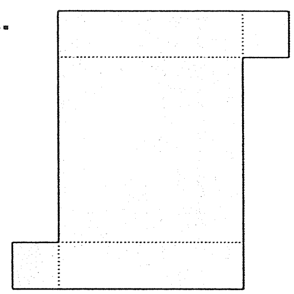
3. 

4. 

5. Which of these nets could be folded along the lines to form a closed rectangular box?

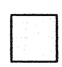
A. 

B. 

C. 

6. Do parts (a)–(c) for each pattern from Exercise 5 that forms a closed rectangular box.

a. Use the unit square shown to help you find the dimensions of the box.

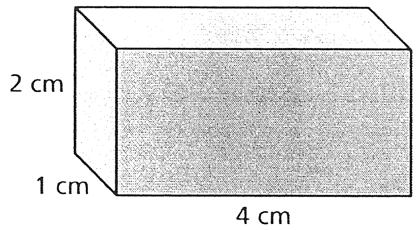


unit square

b. Find the total area, in square units, of all the faces of the box.

c. Find the number of unit cubes it would take to fill the box.

7. This closed rectangular box does not have square ends.



a. What are the dimensions of the box?

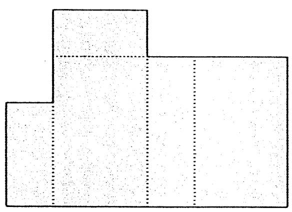
b. On centimeter grid paper, sketch two nets for the box.

c. Find the area, in square centimeters, of each net.

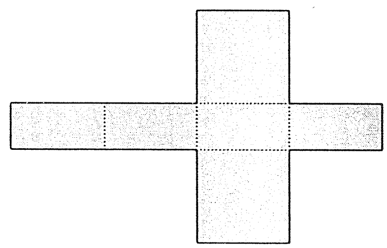
d. Find the total area of all the faces of the box. How does your answer compare with the areas you found in part (c)?

8. Which of these patterns can be folded along the lines to form a closed rectangular box? Explain.

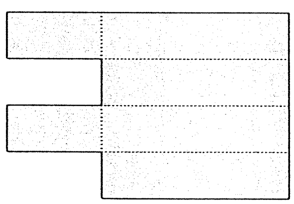
a.



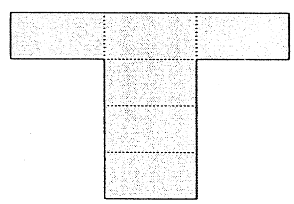
b.



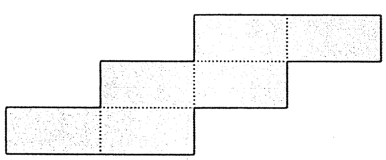
c.



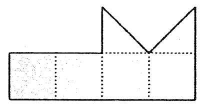
d.



e.



9. Can you fold this net along the lines to form an open cubic box? Explain your reasoning.



For each box described in Exercises 10–13:

- Make a sketch of the box and label the dimensions.
- Draw a net.
- Find the area of each face.
- Find the total area of all the faces.

10. a rectangular box with dimensions
2 centimeters × 3 centimeters × 5 centimeters

11. a rectangular box with dimensions
$2\frac{1}{2}$ centimeters × 2 centimeters × 1 centimeter

12. a cubic box with side lengths $3\frac{2}{3}$ centimeters

13. a cubic box that holds 125 unit cubes

Homework Help Online
PHSchool.com
For: Help with Exercise 10
Web Code: ane-6110

14. An open box is a box without a top.

a. On grid paper, sketch nets for three different open cubic boxes.

b. On grid paper, sketch nets for three different open rectangular boxes (not cubic boxes) with square ends.

c. Find the area of each net you found in parts (a) and (b).

Investigation 1

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 1.1

Core 1–3, 16
Other *Applications* 4, *Connections* 15, 17–18; *Extensions* 32

Problem 1.2

Core 5–7
Other *Connections* 19–24; and unassigned choices from previous problems

Problem 1.3

Core 8–9
Other *Connections* 25–27, 31; and unassigned choices from previous problems

Problem 1.4

Core 10–13
Other *Applications* 14, *Connections* 28–30, *Extensions* 33; and unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 7 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 15, 18–22, 26: *Covering and Surrounding*; 27–29: *Bits and Pieces II*; 30, 31: *Bits and Pieces III*

Applications

1-4. Patterns 2 and 4 *do* form closed boxes. Patterns 1 and 3 *do not*.

5. Patterns A and B *can* be folded to form a closed box. Pattern C *cannot*.

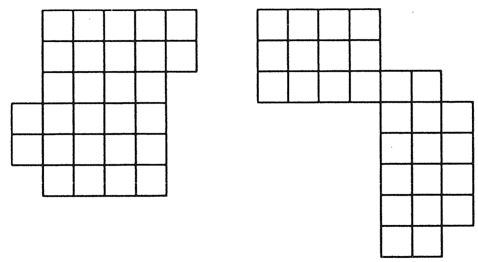
6. **a.** Pattern A: 1 unit by 1 unit by 4 units
Pattern B: 1 unit by 2 units by 4 units

b. Pattern A: 18 sq. units
Pattern B: 28 sq. units

c. Pattern A: 4 cubes
Pattern B: 8 cubes

7. **a.** 2 cm by 4 cm by 1 cm

b. Possible answers:



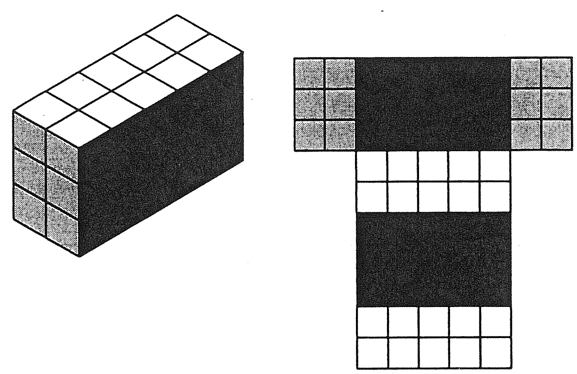
c. All nets for this box have an area of 28 sq. cm.

d. There are two faces with area of 8 sq. cm, two with area 2 sq. cm, and two with area 4 sq. cm, for a total of 28 sq. cm. This is the same as the area of the net.

8. a, c, d, and e *will not* fold into a box. b *will*.

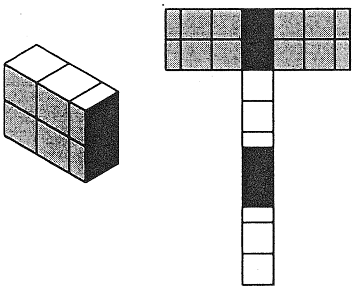
9. This net *will* fold into an open cubic box. The two triangles will meet to become one end of the box

10. Sketch of box and possible net:



There are two of each of these faces:
2 cm by 3 cm (area is 6 sq. cm);
2 cm by 5 cm (area is 10 sq. cm);
3 cm by 5 cm (area is 15 sq. cm.).
The sum of the areas of the faces is 62 sq. cm.

11. Sketch of box and possible net:



There are two of each of these faces:

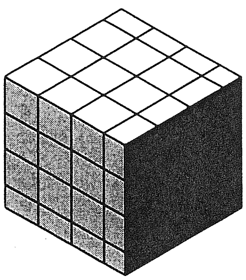
2 cm by 1 cm (area is 2 sq. cm);

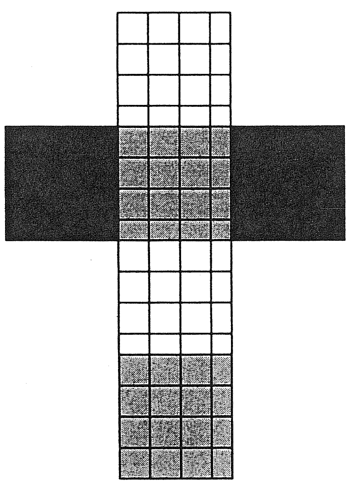
2 cm by $2\frac{1}{2}$ cm (area is 5 sq. cm);

1 cm by $2\frac{1}{2}$ cm (area is $2\frac{1}{2}$ sq. cm).

The sum of the areas of the faces is 19 sq. cm.

12. Sketch of box and possible net:

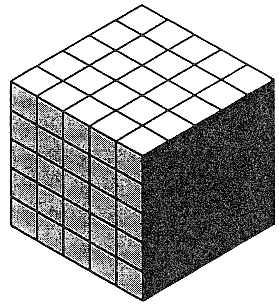


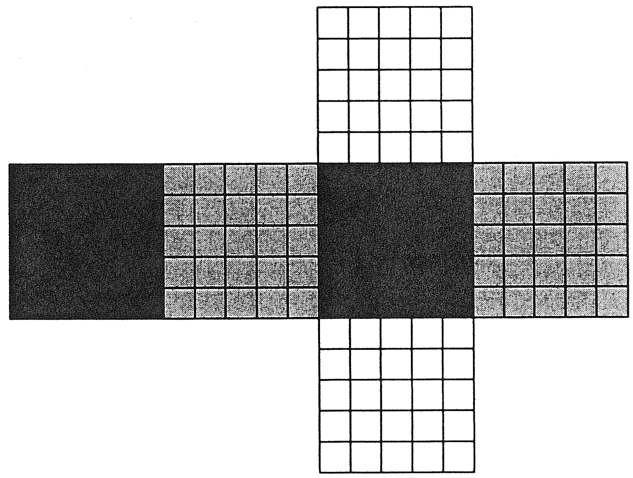


There are six faces. Each is $3\frac{2}{3}$ cm by $3\frac{2}{3}$ cm (each face has area $13\frac{4}{9}$ sq. cm).

The sum of the areas of the faces is $80\frac{2}{3}$ sq. cm.

13. Sketch of box and possible net:

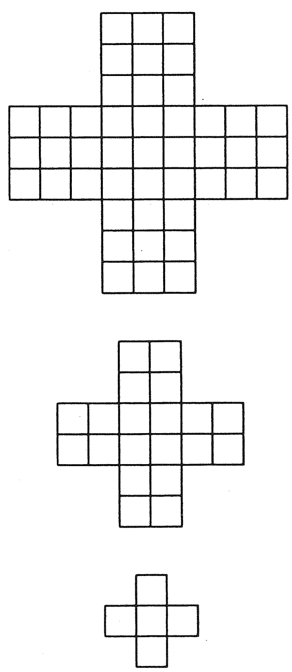




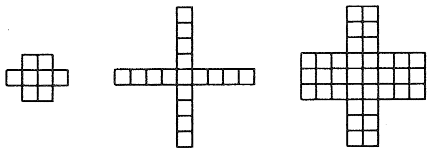
There are six faces. Each is 5 cm by 5 cm (area is 25 sq. cm).

The sum of the areas of the faces is 150 sq. cm.

14. a. Possible nets:



b. Possible nets:



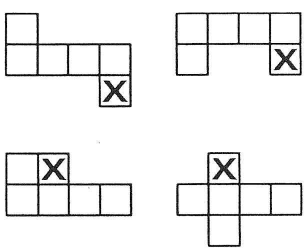
c. Answers will depend on answers to parts (a) and (b). For the examples given above, the areas (in order) are: 45 sq. units, 20 sq. units, 5 sq. units, 8 sq. units, 13 sq. units, and 36 sq. units.

Connections

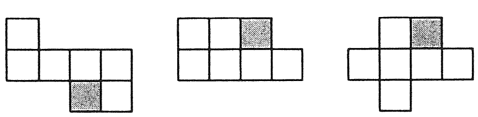
15. A, B, C, and E all have perimeter 14 units; D has perimeter 12 units.

16. Hexominos B and E can be folded to form a closed, cubic box.

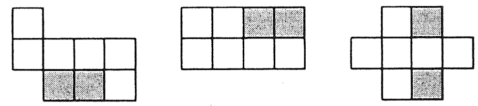
17. Any of hexominos B, C, D, or E can have one square removed to form a net for an open cubic box. Examples:



18. a. Hexominos B, C, D, and E can all have one square added while maintaining the same perimeter. The perimeter does not change if we tuck the new square into a corner—the square covers two units of perimeter while adding two new units. Example at right (the shaded square has been added in each case):



b. Hexominos B, D, and E can have two squares added while maintaining the same perimeter. Examples below:



19. area = 22 cm^2
perimeter = 20 cm

20. area ≈ 28.27 cm^2
perimeter ≈ 18.85 cm

21. area = 30.59 cm^2
perimeter = 26 cm

22. area = 18 $in.^2$
perimeter = 17.5 in.

23. b, $\angle r$ and $\angle q$

24. n measures 102°

25. A

26. a. To find the area and perimeter of a rectangle, you need to know the length and the width. (In fact, if we consider the set of area, perimeter, length, and width, knowing any two is sufficient information for finding the other two.) To find the area, multiply length by width. To find perimeter, add the length and width, then double the result. An alternate way to find perimeter is to double the length, double the width, and add these two results.

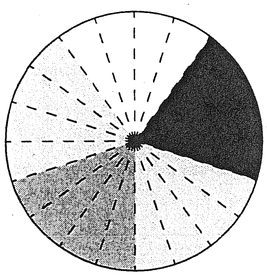
b. Since, in a square, length and width are equal, you need only to know the length of a side. To find the area, multiply the side length by itself. To find the perimeter, multiply the side length by four.

27. a. $\frac{11}{12} \div \frac{1}{8} = 7\frac{1}{3}$. Ms. Zhou can make slats for 7 doll beds and have enough left (if it is usable) for $\frac{1}{3}$ of another bed.

b. (Figure 1)

28. a. $\frac{3}{5} \div 4 = \frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$. Each gets $\frac{3}{20}$ of the pie.

b.



29. a. $3\frac{1}{2} \div \frac{3}{8} = \frac{28}{8} \div \frac{3}{8} = 28 \div 3 = 9\frac{1}{3}$ recipes

b. (Figure 2)

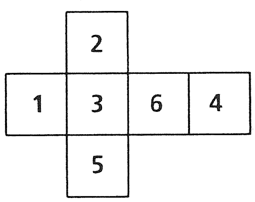
30. $0.65 \div 0.15 = 4.33$ scoops.

31. 2,500 ml

$$0.16\text{ c} = 400\text{ ml}$$
$$\text{c} = 400 \div 0.16$$
$$\text{c} = 2{,}500\text{ ml}$$

Extensions

32. Possible answer:



33. Several shapes will allow 12 boxes to be made (for example, Figure 3 below). At least one shape will allow 13 boxes (for example, Figure 4 below).

Figure 3

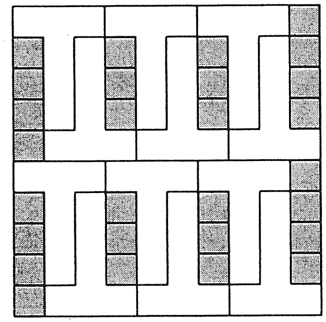


Figure 4

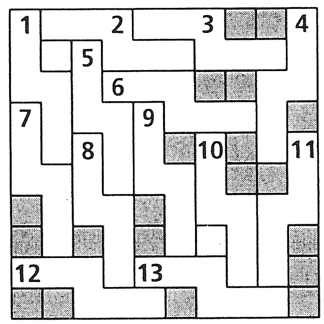


Figure 1

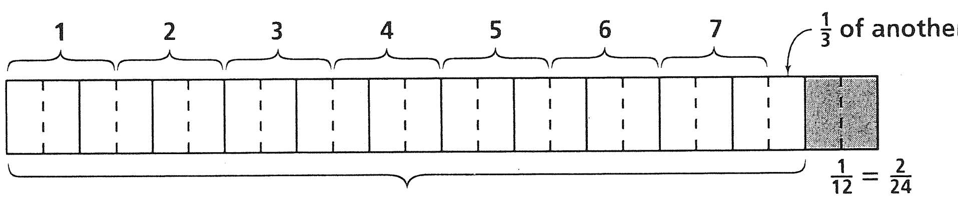
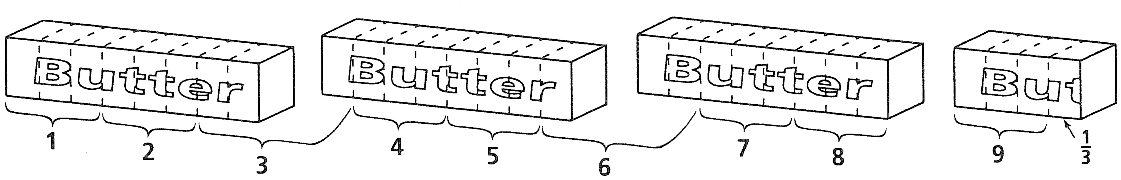


Figure 2



Possible Answers to Mathematical Reflections

1. The faces of a rectangular box are rectangles. You just need to find the surface area of each rectangular face and add these areas. Because every box has three matching pairs of faces, you could find the area of the three different faces and then double this total.
2. You can find the number of cubes it would take to fill a box by putting the cubes inside the box and counting how many fit. If the cubes do not fit exactly, you have to estimate the partial cubes that are needed. Some students may begin to see a rule, but do not push for it at this point.
3. The number of square units in the net must be the same. The arrangement of the square units can be different, as can be the perimeter of the net. (Note: Students may have other observations. You might want to discuss students' reflections and test their ideas by examining some of the nets from this investigation.)

Mathematical Reflections 1

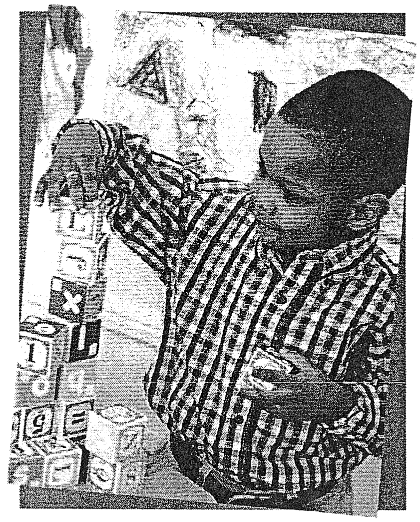
In this investigation, you explored rectangular boxes, and you made nets for boxes. You found the dimensions of a box, the total area of all its faces, and the number of unit cubes required to fill it. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Explain how to find the total area of all the faces of a rectangular box.
2. Explain how to find the number of identical cubes it will take to fill a rectangular box.
3. Suppose several different nets are made for a given box. What do all of the nets have in common? What might be different?

2.1 Packaging Blocks

ATC Toy Company is planning to market a set of children's alphabet blocks. Each block is a cube with 1-inch edges, so each block has a volume of 1 cubic inch.



Problem 2.1 Finding Surface Area

The company wants to arrange 24 blocks in the shape of a rectangular prism and then package them in a box that exactly fits the prism.

A. Find all the ways 24 cubes can be arranged into a rectangular prism. Make a sketch of each arrangement. Record the dimensions and surface area. It may help to organize your findings into a table like the one below:

Possible Arrangements of 24 Cubes

Length	Width	Height	Volume	Surface Area	Sketch
▒	▒	▒	▒	▒	▒
▒	▒	▒	▒	▒	▒
▒	▒	▒	▒	▒	▒

B. Which of your arrangements requires the box made with the least material? Which requires the box made with the most material?

C. Which arrangement would you recommend to ATC Toy Company? Explain why.

D. Why do you think the company makes 24 alphabet blocks rather than 26?

ACE Homework starts on page 24.

2.1 Packaging Blocks

Goals

- Connect the dimensions of a rectangular prism to its volume and surface area
- Understand that rectangular prisms may have the same volume but quite different surface areas

In Investigation 1, students were introduced to the idea of the surface area of a rectangular box and should have begun to make connections between the surface area and the dimensions of a box. In this problem, they find all the possible rectangular arrangements of 24 blocks and the amount of material needed to package them. Students may still be focusing on the area of each face of a box, but they should be using their knowledge about finding the area of a rectangle rather than counting individual squares. As they find different arrangements for the 24 blocks, they also begin to see connections between the dimensions of the box and its volume.

You may have introduced the vocabulary of surface area and volume in Investigation 1. If not, it can be introduced in the Launch.

Launch 2.1

Use the nets and boxes from Investigation 1 to demonstrate the concepts of volume and surface area.

Suggested Questions Ask:

- *How many unit cubes fit inside of box R?* (16)
- *The word for the number of unit cubes that fill a solid is volume. So the volume of box R is 16 cubic units. What is the volume of box S?* (also 16)
- *So boxes R and S have the same volume. What is different about these two boxes?* (the shape, the height, the areas of their nets)
- *We saw that the area of the net was the same as the sum of the areas of the faces of the boxes. We call this sum the* surface area *of the solid. Which box has a larger surface area, R or S?* (R has a surface area of 40 sq. units; S has a surface area of 48 sq. units, so S has a larger surface area).

A net illustrates surface area in a way that students are apt to remember: to find the total surface area, we find the area of each face and add them.

Some students will struggle with the transition between the net and the box. They may need to see the net fold into the box many times and think about the relationship between the area of the net and the total area of the faces of the box each time. Be alert for these struggles in the next few problems.

Tell the story of ATC Toy Company. Before students break into groups, ask the class to suggest one arrangement of 24 blocks and discuss how they might find its surface area. If you want students to organize their data in a table (as shown in the student edition), model the process by entering the data about the chosen arrangement into a table. Or, let students decide how to organize their work to look for patterns.

You may suggest that students make sketches for only one or two of the boxes. As discussed in the Introduction to the unit, isometric dot paper may be very helpful for students in making their sketches.

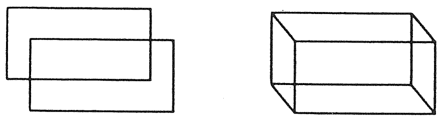
Have students work in groups of two to four. Distribute unit cubes (inch cubes, if you have them) to each group.

Explore 2.1

Encourage students to organize their information in a table as suggested in the problem or in some other way that makes sense to them. They should sketch each arrangement they find and label its dimensions. Encourage students to find ways to ensure that they have found all the boxes with whole-number dimensions.

As you listen to students talk and ask them questions, encourage the use of the vocabulary: surface area and volume.

Visualizing how to sketch the boxes may be difficult for some students. One technique is to think of drawing two offset rectangles, then connecting the corners to form the box.



Summarize 2.1

Begin the summary by collecting the data students recorded in their tables. You might start with the 24-by-1-by-1 box and model collecting data in an organized manner.

Suggested Questions Ask:

- *Did anyone find a box that holds exactly 24 cubes and has an edge length of 1? What is the length of the base of this box? What is the width of the base of this box? What is the height of this box?*
- *You know this box has a volume of 24 cubic inches because that was a requirement. How much material will it take to cover this box?*
- *Did anyone find a box that holds exactly 24 cubes and has an edge length of 2? What is the length of the base of this box? What is the width of the base of this box? What is the height of this box?*
- *How much material will it take to cover this box?*

Continue with this line of questioning for edge lengths of 3, 4, 5, 6, 7, 8, 12, and 24. Asking for an edge length of 5 or 7 should give rise to a discussion about factors.

You may need to discuss suggested arrangements that are identical; for example, someone may suggest the arrangement with length 2, width 4, and height 3 and another the arrangement with length 4, width 3, and height 2. To demonstrate their equivalence, build the arrangement and set it on the three possible bases. The edges chosen to be length, width, and height are arbitrary, although it is customary to use the length and width of the base as the length and width of the rectangular box.

Students' sketches will vary, depending on which face they use as the base.

- *How did you decide which face to use for the base? Does your choice affect the surface area of the box?* (no)

Suggested Question When you have collected all the arrangements that were found, ask students to describe the patterns they see in the table.

- *Look at the table we have generated. What patterns do you notice? Explain why the patterns make sense.*

Here are some patterns students have noticed:

Classroom Dialogue Model

Patterns From the Table

Chandra: *"The volume is always 24 cubic inches."*

This is a requirement of the problem.

J.J.: *"As one dimension increases, another one decreases."*

Pedro: *"If you put more cubes in the base, the height decreases because the total is still 24."*

Cie: *"Boxes with the same three dimensions have the same volume and surface area; a 1-by-3-by-8 box and an 8-by-3-by-1 box have the same volume and surface area."*

They are really the same box oriented differently.

Ali: *"The product of the length, width, and height must equal 24, which is the volume."*

Since length times width tells how many cubes are in a layer, and height tells how many layers there are, multiplying them will give the number of cubes, or cubic inches, that will fill the box.

Some students will begin to understand that the factors of 24 are what determine the possible arrangements of 24 cubes. Some may see that volume is equal to length × width × height and use this idea to find boxes that work. If students offer the formula for finding the volume of a box, ask them to try the rule on some other boxes—for example, a box with a length of 7 units, a width of 3 units, and a height of 2 units, or a box with a length of 6 units, a width of 8 units, and a height of 1 unit. Ask them why this works. Be sure that students understand the "layering" strategy for finding volume—the number of blocks in the first layer times the number of layers or area of the base times height. Ask them to build these boxes and check to see that the pattern for finding the volume works. If students have not yet discovered this, it will surface in the next problem.

At this point, students will begin to see that, to find the surface area of a prism, they need to find the area of each of the six faces and add them. Some will see that opposite faces are equivalent and will double the area of a face to get the area of the pair.

Students may have difficulty when trying to work from the dimensions alone. For example, the surface area of a 2-by-3-by-4 box can be found from this information alone, as each pair of dimensions specifies two faces of the box (2 by 3, 2 by 4, and 3 by 4). Many students will still need to sketch or build the box or make a pattern for it to find the surface area. Asking them to notice opposite faces will move them toward a more efficient process for determining surface area.

Suggested Questions Discuss in detail which box has the least surface area (requires the least amount of material), which has the greatest, and what these boxes look like.

- *Which of the boxes with a volume of 24 cubic units has the greatest surface area?* (the 1-by-1-by-24 box)
- *What does it look like?* (long and skinny)
- *Which has the least surface area?* (the 2-by-3-by-4 box)
- *What does it look like?* (more like a cube)
- *If you were going to make a box to hold 36 cubes, which of the possible arrangements of 36 cubes would have the greatest surface area?* (a 1-by-1-by-36 arrangement, with a surface area of 146 square units)
- *Why?* (because the cubes are spread out as much as possible)
- *If you were going to make a box to hold 36 cubes, which design would cost you the least to enclose?* (a 3-by-3-by-4 arrangement, with a surface area of 66 square units)
- *Why?* (because the cubes are arranged in a more compact fashion, so more faces of the cubes are covered up)

2.1 Packaging Blocks

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goals

- Connect the dimensions of a rectangular prism to its volume and surface area
- Understand that rectangular prisms may have the same volume but quite different surface areas

Launch

Use the nets and boxes from Investigation 1 to demonstrate the concepts of volume and surface area.

- *How many unit cubes fit inside of box R?*
- *The word for the number of unit cubes that fill a solid is* volume. *What is the volume of box S?*
- *So boxes R and S have the same volume. What is different about these two boxes?*
- *We call the sum of the areas of the faces the* surface area *of the solid. Which box has a larger surface area, R or S?*

Tell the story of ATC Toy Company. Before students break into groups, have the class suggest one arrangement of 24 blocks and discuss how they might find its surface area. If you want students to organize their data in a table (as shown in the student edition), model the process by entering the data about the chosen arrangement into a table. Or, let students decide how to organize their work to look for patterns.

You may suggest that students make sketches for only one or two of the boxes.

Have students work in groups of two to four.

Materials

- Inch cubes or other unit cubes
- Transparency 2.1
- Nets and boxes from Investigation 1

Vocabulary

- volume
- surface area

Explore

Encourage students to organize their information in a table as suggested in the problem or in some other way that makes sense to them. They should sketch each arrangement they find and label its dimensions.

As you listen to students talk and ask them questions, encourage the use of the vocabulary: surface area and volume.

Summarize

Begin the summary by collecting students' data.

- *Did anyone find a box that holds exactly 24 cubes and has an edge length of 1?*
- *How much material will it take to cover this box?*
- *Did anyone find a box that holds exactly 24 cubes and has an edge length of 2?*

Continue with this line of questioning. Discuss identical arrangements.

Materials

- Student notebooks

continued on next page

Summarize
continued

- *How did you decide which face to use for the base? Does your choice affect the surface area of the box?*

Have students describe the patterns they see in the table.

Discuss which box has the least surface area, which has the greatest, and what these boxes look like.

ACE Assignment Guide for Problem 2.1

Differentiated Instruction
Solutions for All Learners

Core 1–3, 20
Other *Connections* 21, 22

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 20: *Prime Time*; 22: *Variables and Patterns*

Answers to Problem 2.1

A. (Note: Students' sketches may show the same arrangement in a different orientation.)

(Figure 1)

B. The 4-by-3-by-2 box requires the least amount of material. The 24-by-1-by-1 box requires the most material.

C. Possible answer: ATC Toy Company should use the 4-by-3-by-2 box because it has the least surface area (52 in.^{2}) and would therefore be the least expensive to buy or to make. (Note: The box shaped most like a cube will always have the least surface area. This is pursued in more depth in Problem 2.2. Don't expect your class to make this generalization at this time.)

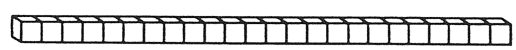
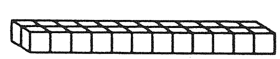
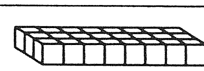
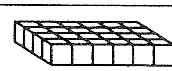
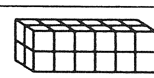
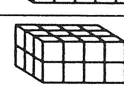
Some students may argue for boxes based on their visual appeal to the buyer.

D. Possible answer: Because 24 has more factors than 26, there are more ways to effectively package 24 blocks. With 26 blocks, you only have a 1-by-1-by-26 or a 1-by-2-by-13 arrangement. These two boxes are long and thin, so they will have larger surface areas than if the box could be more cubic in shape, like you can get with 24 blocks.

Some students might argue that the lesser-used letters don't need their own blocks, so the company can economize by making sets of 24 instead of 26.

Figure 1

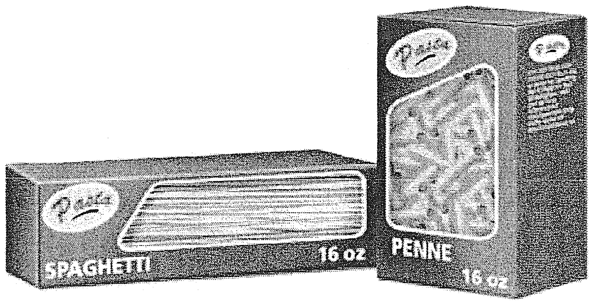
Possible Arrangements of 24 Cubes

Length (in.)	Width (in.)	Height (in.)	Vol (in.^{3})	Surface areas (in.^{2})	Sketch
24	1	1	24	98	
12	2	1	24	76	
8	3	1	24	70	
6	4	1	24	68	
6	2	2	24	56	
4	3	2	24	52	

2.2 Saving Trees

You discovered that 24 blocks can be packaged in different ways that use varying amounts of packaging material. By using less material, a company can save money, reduce waste, and conserve natural resources.

Which rectangular arrangement of cubes uses the least amount of packaging material?



Problem 2.2 Finding the Least Surface Area

A. Explore the possible arrangements of each of the following numbers of cubes. Find the arrangement that requires the least amount of packaging material.

1. 8 cubes **2.** 27 cubes **3.** 12 cubes

B. **1.** Make a conjecture about the rectangular arrangement of cubes that requires the least packaging material.

2. Does your conjecture work for 30 cubes? Does it work for 64 cubes? If not, change your conjecture so it works for any number of cubes. When you have a conjecture that you think is correct, give reasons why you think your conjecture is valid.

C. Describe a strategy for finding the total surface area of a closed box.

ACE Homework starts on page 24.

2.2 Saving Trees

Goals

- Predict which rectangular prism of those with a common volume will have the smallest surface area
- Refine a strategy for finding the surface area of a rectangular prism

This problem encourages students to find a general pattern for which rectangular arrangement of a given number of cubes will have the least surface area. The summary of Problem 2.1 leads nicely into changing the context from looking for a box that will hold exactly 24 cubes to investigating whether there is a way to find the box with minimal surface area for any fixed volume.

Launch 2.2

Review what students learned in Problem 2.1.

- *How would you describe the shape of the box we found in the last problem that held 24 cubes and had the least amount of surface area?* (The box was the one in which the dimensions were the closest, 4 by 3 by 2.)

Introduce Problem 2.2

It took a lot of work to find all the possible box arrangements for a volume of 24 cubic units. From the table we made, we found the box with the least surface area.

In mathematics, we are always looking for patterns and rules that will help us to predict outcomes. In today's problem, you are challenged to explore rectangular prisms with different volumes. You are asked to look carefully at the data and make conjectures about what you think will help you to predict the arrangement with the smallest surface area.

Let the class work on the problem in groups of 3 or 4.

Explore 2.2

Encourage groups who make conjectures about the arrangement of cubes that requires the least amount of packaging material to test other arrangements of the same number of cubes.

- *Test your conjectures on a number of cubes other than the 8, 27, and 12 suggested in the problem.*

Groups will have to find a way to organize their data, probably by using a table. If a group is having trouble with the problem, talk through the case of eight cubes with them. Ask them to build each arrangement and to look at the physical objects as well as the measures in their table.

Suggested Questions Ask:

- *Look at the dimensions for each arrangement and how they change from one arrangement to another. What is the difference between the box with the greatest surface area and the box with the least surface area?*
- *How does this difference show up in the actual boxes made from cubes?*
- *How does this difference show up in the dimensions of the boxes?*

Going Further Suppose we can fill the box with non-whole cubes (parts of cubes). How would this change your answers to Problems 2.1 and 2.2?

Summarize 2.2

Ask students to explain why the more cube-like rectangular arrangement requires the least packaging material.

Describe how you found the amount of packaging material (the surface area) required for the different arrangements you made.

- *What are the dimensions of the box with the least surface area that holds 8 cubes? The greatest surface area?*

Suggested Questions Ask the same questions for 27 cubes and 12 cubes. Display the answers to the three questions, putting the boxes with the greatest surface area together and those with the least surface area together.

- *How would you describe these shapes compared to these?* (Those with the greatest surface area are long and spread out; those with the least surface area are more compact, more like a cube.)
- *Why is the more cube-like rectangular box the box with the least surface area?* (Students might answer this by saying something like, "The cube shape hides some squares inside, so their faces do not get counted in the surface area of the cube. In the long 1-by-1-by-27 arrangement, all the cubes have faces exposed; in the 3-by-3-by-3 arrangement, one cube is completely hidden.")

Have students describe their processes for finding surface area. Students should be at a point at which it is appropriate for you to model a symbolic representation of their strategies. For instance, if a student says something like:

> *"We found the area of the front, the area of the top, and the area of the right side, then doubled the total."*

You could write $(w \times h + w \times \ell + \ell \times h) \times 2$. This is not necessarily to develop a formula that students need to memorize. (Formulas for surface area are complicated, and often more easily reconstructed from a visual or image of the surface area of the prism as the area of the six faces of the prism than memorized.) Instead, it is to encourage the kind of careful thinking required to write a formula.

Check for Understanding

Have students describe the dimensions of the box with the least surface area, and then with the greatest surface area, for 100 cubes.

Then, help the class further explore the minimal surface area.

- *For 12 cubes, you found the arrangement with the least surface area to be a 2-by-2-by-3 box. If you could cut the cubes apart, could you package the same volume with even less surface area?* (The arrangement with the least surface area for 8 and 27 cubes is a cubic box. The 12-cube arrangement raises the issue of whole-number edges versus fractional-length edges for the minimum surface area.)

Students may suggest something like the following: "It's a cube whose dimensions are all the same but when you multiply them together, they equal 12"—in other words, the cube root of the volume. If this comes up, you can use a calculator to guess and check for this number, which is approximately 2.289 or 2.29.

Going Further You may want to ask the class to compare the shapes of animals that live in cold climates to those that live in the desert—for example, a polar bear and a snake. Polar bears are more cube-like in that they are designed with a small surface area compared to their volume. This minimizes the escape of the heat in their warm blood to the cold atmosphere. Snakes have a great deal of surface area compared to their volume, which allows them to quickly use the sunshine to heat their cold blood.

At a Glance

2.2 Saving Trees

PACING 1 day

Mathematical Goals

- Predict which rectangular prism of those with a common volume will have the smallest surface area
- Refine a strategy for finding the surface area of a rectangular prism

Launch

Materials
- Inch cubes
- 2 or 3 rectangular or cubic boxes

Review what students discovered in Problem 2.1.

- *How would you describe the shape of the box we found in the last problem that held 24 cubes and had the least amount of surface area?*

Introduce Problem 2.2.

In mathematics, we are always looking for patterns and rules that will help us to predict outcomes. In today's problem, you are challenged to explore prisms with different volumes. You are asked to look carefully at the data and make conjectures about what you think will help you to predict the arrangement with the smallest surface area.

Let the class work on the problem in groups of 3 or 4.

Explore

Encourage groups who make conjectures about the arrangement of cubes that requires the least amount of packaging material to test other arrangements of the same number of cubes.

Test your conjectures on a number of cubes other than the 8, 27, and 12 suggested in the problem.

If a group is having trouble with the problem, talk through the case of eight cubes with them. Ask them to build each arrangement and to look at the physical objects as well as the measures in their table.

Look at the dimensions for each arrangement and how they change from one arrangement to another.

- *What is the difference between the box with the greatest surface area and the box with the least surface area?*
- *How does this difference show up in the actual boxes made from cubes?*
- *How does this difference show up in the dimensions of the boxes?*

Summarize

Materials
- Student notebooks

Ask students to explain why the more cube-like rectangular arrangement requires the least packaging material.

Put the boxes with the greatest surface area together and those with the least surface area together.

- *How would you describe these shapes compared to these?*

continued on next page

Summarize *continued*

- *Why is the more cube-like rectangular box the box with the least surface area?*

Have students describe their processes for finding surface area. Model a symbolic representation of their strategies. This is not to develop a formula that students need to memorize. Instead, it is to encourage the kind of careful thinking required to write a formula.

Help the class further explore the minimal surface area.

ACE Assignment Guide for Problem 2.2

Differentiated Instruction
Solutions for All Learners

Core 4–6
Other *Connections* 23–24; *Extension* 28; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 5 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 23–24: *Covering and Surrounding*

Answers to Problem 2.2

A. The rectangular arrangements of cubes with the least surface area are:

1. 8 cubes: 2 by 2 by 2 (surface area: 24 in.^{3})
2. 27 cubes: 3 by 3 by 3 (surface area: 54 in.^{3})
3. 12 cubes: 2 by 2 by 3 (surface area: 32 in.^{3})

B. 1. Possible true conjecture: The rectangular arrangement of a given number of cubes with the least surface area is the one that is most like a cube. Students may also use language like *compact* or *shortest* (in contrast to the long, skinny packages). This is similar to the conjecture that students examined in *Covering and Surrounding*: that the rectangle with the smallest perimeter for a given area is a square.

2. The conjecture in B gives:

30 cubes: 2 by 3 by 5

64 cubes: 4 by 4 by 4, each of which has the smallest surface area for the given number of cubes.

One way to think about justifying the conjecture is that the exposed faces of the small cubes generate the surface area. The more compact (or cube-like) the prism, the more faces of the small cubes face the interior of the prism, so fewer are exposed as surface area.

C. Possible answers: Find the area of each of the six faces and add them together. Find the area of the front, the top, and the right side; add these together and double the answer.

Did You Know?

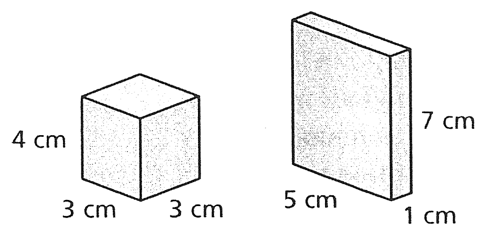
Area is expressed in square units, such as square inches or square centimeters. You can abbreviate square units by writing the abbreviation for the unit followed by a raised 2. For example, an abbreviation for square inches is in.^{2}.

Volume is expressed in cubic units. You can abbreviate cubic units by writing the abbreviation for the unit followed by a raised 3. For example, an abbreviation for cubic centimeters is cm^3.

Getting Ready for Problem 2.3

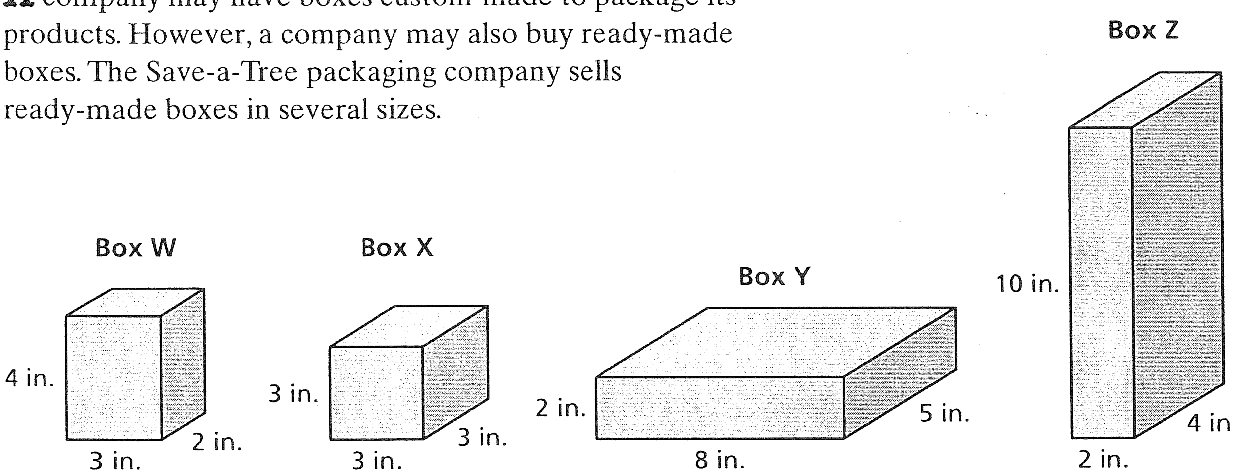
One seventh-grade student, Bernie, wonders if he can compare volumes without having to calculate them exactly. He figures that volume measures the contents of a container. He fills the prism on the left with rice and then pours the rice into the one on the right.

- How can you decide if there is enough rice or too much rice to fill the prism on the right?



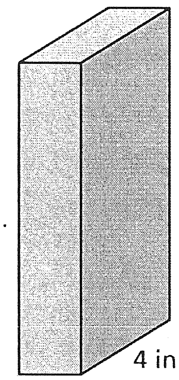
2.3 Filling Rectangular Boxes

A company may have boxes custom-made to package its products. However, a company may also buy ready-made boxes. The Save-a-Tree packaging company sells ready-made boxes in several sizes.



Problem 2.3 Finding the Volume of a Rectangular Prism

ATC Toy Company is considering using Save-a-Tree's Box Z to ship alphabet blocks. Each block is a 1-inch cube. ATC needs to know how many blocks will fit into Box Z and the surface area of the box.

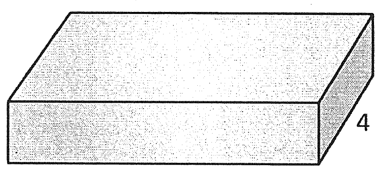


A. The number of unit cubes that fit in a box is the volume of the box.

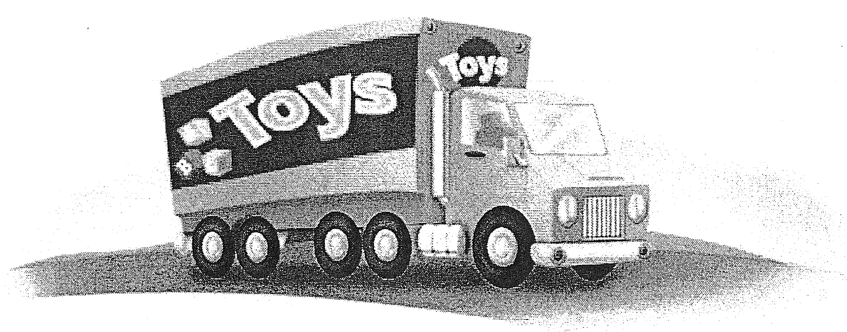
1. How many cubes will fit in a single layer at the bottom of this box?
2. How many identical layers can be stacked in this box?
3. What is the total number of cubes that can be packed in this box?
4. Consider the number of cubes in each layer, the number of layers, the volume, and the dimensions of the box. What connections do you see among these measurements?

B. Find the surface area of Box Z.

C. Suppose Box Z is put down on its side so its base is 4 inches by 10 inches and its height is 2 inches. Does this affect the volume of the box? Does this affect the surface area? Explain your reasoning.



D. Apply your strategies for finding volume and surface area to Boxes W, X, and Y.



ACE Homework starts on page 24.

2.3 Filling Rectangular Boxes

Goals

- Understand that prisms can be filled systematically in identical layers, and that this layering leads to the formula for volume
- Develop a formula for finding the volume of a rectangular prism

In this problem, students are helped to think about how to fill the prism systematically. First, they place one layer of cubes on the base. The number of cubes is equal to the area of the base—each cube (or part of a cube) rests on a square (or part of a square) in the base of the prism, so there is a one-to-one correspondence between the number of cubes and the area of the base. Then, they determine how many layers of cubes are needed to fill the box. This is equal to the height of the box. Thus, the volume of the box is the number of cubes in the bottom layer multiplied by the number of layers—the area of the base times the height of the prism.

In a later investigation, students will study cones and spheres. For these shapes, layering is not a useful strategy for thinking about volume. Instead, students will fill these shapes with clay or rice, then pour the contents into a container of known volume.

Launch 2.3

Discuss the Getting Ready. This is an opportunity to help your students think a little bit differently about volume.

Bernie is thinking correctly about volume. There will be a little too much rice for the prism on the right because its volume is 35 cm^3 while the volume of the other prism is 36 cm^3. Students may not know the exact volume, but they may suggest filling one box with rice and pouring it into the other box.

It is helpful at this stage to know whether your students see layers and filling with rice as two ways to measure the same thing. If they do not, you may want to take time to have them fill rectangular prisms with rice and make comparisons of their own.

Talk about Save-a-Tree's ready-made box sizes and ATC Toy Company's decision. Hold up a box.

- *What is the volume of my box? How did you make your estimate?*

Constructing one or all of the boxes from transparent grids might help students visualize the process of finding volume. If you have the time to make the transparent boxes, show them to the class or distribute one box to each pair of students. Ask students to estimate the volume of each box. Record some of the estimates on the board. Tell the class that the intent of this problem is for them to look for efficient ways to find the volume of a box. If some students claim that they already have a rule for finding the volume of a box (volume = $\ell \times w \times h$), question them about it.

Suggested Questions Ask:

- *What does your rule mean?*
- *Why do you think it will work?*
- *Will it work for all prisms?*

Students can work in pairs.

Explore 2.3

Remind students to save the transparent boxes for the summary. Some students may need cubes to simulate filling the boxes.

You may want to suggest that students organize their work in a table. The organization of the following table will help promote the layering strategy for determining volume.

Box	Cubes in a Single Layer	Number of Identical Layers	Volume	Surface Area (optional)
W				
X				
Y				
Z				

As students make progress in their pairs, ask them how close their estimates of the volume were to the answers they are finding.

Summarize 2.3

Discuss the answers to Question A. At this point in the unit, two popular strategies for finding the number of blocks that will fill a box are the following:

"First we found a layer for box W. The base is 2 inches by 3 inches, so it takes 6 cubic inches to form one layer. Then we saw that it would take four of these layers to fill the box."

"We used layering, but we saw that for box W, 3×2 is the number of blocks in one layer, and $3 \times 2 \times 4$ is the number of blocks in four layers. We think you can just multiply the three dimensions to get the volume of the box."

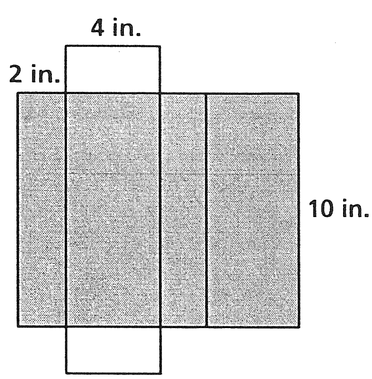
Suggested Questions If some students offer the formula volume $= \ell \times w \times h$, ask:

- *What does* $\ell \times$ w $\times$ h *mean in terms of counting layers?* [$\ell \times w$ is the area of the base (the number of cubes in the first layer) and h is the height (number of layers). The strategy of multiplying the area of the base by the height will work for all prisms and cylinders.]

Talk about the answers to Question B. Students may offer these strategies for finding the surface area of a box:

"We saw that for each box, two faces are the same. So we found the area of each of the three different faces and multiplied each one by 2. Then we just added the three numbers."

"We pictured folding the box Z flat like this:



"We saw that we had one big rectangle (shaded) that is 12 by 10 and two small rectangles that are 2 by 4. So, the surface area is $12 \times 10 + 2 \times 2 \times 4$."

The following questions can be used if needed to focus students' attention on the bottom layer of a prism and how many layers it will take to fill the prism.

- *Why is the number of cubes in the bottom layer equal to the area of the base?* (Each square unit of area can be thought of as the base of a unit cube.)

If you have constructed models of the boxes, hold one of them up at the orientation shown in the problem.

- *Make a sketch of this box while I sketch it at the overhead.*
- *What are the dimensions of the base of this box? What is its height? What is the volume of the box?*
- *Now, set the box on a different base.*
- *What is the area of the new base? How many cubes will fit on the base?*
- *How many layers will be needed to fill the box?*
- *Does this new orientation change the volume? Explain your answer.*
- *What is the surface area of this box? Would changing the orientation of the box change its surface area?*

Students should visualize the surface area as the area of the six faces. This provides them with a strategy that will generalize to all prisms. They find the six areas and add them. Some will notice that in rectangular prisms there are three pairs of congruent faces (opposite faces are congruent). This would shorten their calculations slightly. They would find the area of three faces—one from each pair—and multiply by 2.

Give each pair of students a small box and ask them to find its dimensions and volume. Have them check their answers by filling the box with unit cubes. Or you can use the boxes from Problem 1.4 as a quicker assessment.

For Question D, repeat the questions in Questions A and B of Problem 2.3 for Box W, Box X, and Box Y.

2.3 Filling Rectangular Boxes

At a Glance

PACING 1 day

Mathematical Goals

- Understand that prisms can be filled systematically in identical layers, and that this layering leads to the formula for volume
- Develop a formula for finding the volume of a rectangular prism

Launch

Discuss the Getting Ready.

Talk about Save-a-Tree's ready-made box sizes and ATC Toy Company's decision. Hold up a box.

- *What is the volume of my box? How did you make your estimate?*

Ask students to estimate the volume of each box. Record some of the estimates on the board. Tell the class that the intent of this problem is for them to look for efficient ways to find the volume of a box. If some students claim that they already have a rule for finding the volume of a box (volume $= \ell \times w \times h$), question them about it.

- *What does your rule mean?*
- *Why do you think it will work?*
- *Will it work for all prisms?*

Students can work in pairs.

Materials

- Pre-made box for demonstration
- Transparent grids (optional)
- Transparent models of Boxes W, X, Y, and Z (optional)
- Transparencies 2.3A and 2.3B

Explore

Remind students to save the transparent boxes for the summary. Some students may need cubes to simulate filling the boxes.

You may want to suggest that students organize their work in a table.

As students make progress in their pairs, ask them how close their estimates of the volume were to the answers they are finding.

Materials

- Inch cubes
- Inch or other grid paper

Summarize

Discuss the answers to Question A. If some students offer the formula volume $= \ell \times w \times h$, ask what this means in terms of counting layers. Talk about the answers to Question B. Spend time on students' strategies for finding the surface area of a box.

Ask some questions to probe students' understanding.

- *Why is the number of cubes in the bottom layer equal to the area of the base?*

If you have constructed models of the boxes, hold one of them up at the orientation shown in the problem.

- *What are the dimensions of the base of this box? What is its height? What is the volume of the box?*

Materials

- Student notebooks
- Transparency 2.3C

continued on next page

Summarize *continued*

Now, set the box on a different base.

- *What is the area of the new base? How many cubes will fit on the base? How many layers will be needed to fill the box?*
- *Does this new orientation change the volume? What is the surface area of this box? Would changing the orientation of the box change its surface area?*

Go over the volumes and surface areas of the other boxes.

ACE Assignment Guide for Problem 2.3

Differentiated Instruction
Solutions for All Learners

Core 8–15

Other *Applications* 7, 16–19; *Connections* 25–27; *Extensions* 29; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 2.3

A. **1.** 8 cubes

2. 10 layers

3. 80 cubes

4. See the Summarize section for some possible explanations.

B. 136 in.^{2}

C. The volume doesn't change. The number of cubes in the first layer changes, but so does the number of layers. The volume is: area of base $\times$ height, or $10 \times 4 \times 2$. In the original position, the volume was $2 \times 4 \times 10$. Similarly, the surface area of the box does not change.

D. Surface area: Box W: 52 in.^{2}; Box X: 54 in.^{2}; Box Y: 132 in.^{2}

Volume: Box W: 24 cubes; Box X: 27 cubes; Box Y: 80 cubes

Mathematical Reflections 2

In this investigation, you arranged cubes in the shape of rectangular prisms, and you also found the arrangements with the least and greatest surface area. You developed methods for finding surface area and volume. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. For a given number of cubes, what arrangement will give a rectangular prism with the least surface area? What arrangement will give a rectangular prism with the greatest surface area? Use specific examples to illustrate your ideas.
2. Describe how you can find the surface area of a rectangular prism. Give a rule for finding the surface area.
3. Describe how you can find the volume of any prism. Give a rule for finding the volume.

Name ______________________________ Date ____________________ Period ______

Discovering Surface Area of a Pyramid -- 6th grade Filling and Wrapping

A. Describe the faces of each pyramid flat pattern.

Square Pyramid:	Name the polygons that form the pyramid's flat pattern. What are the dimensions of each face?
Triangular Pyramid:	Name the polygons that form the pyramid's flat pattern. What are the dimensions of each face?
Rectangular Pyramid:	Name the polygons that form the pyramid's flat pattern. What are the dimensions of each face?

B. Find the surface area (total area of the faces) for each pyramid net. Show your work!

Square Pyramid:
Triangular Pyramid:
Rectangular Pyramid:

C. Now cut out your flat patterns and construct each pyramid. Do the pyramids have the same surface area as their nets?

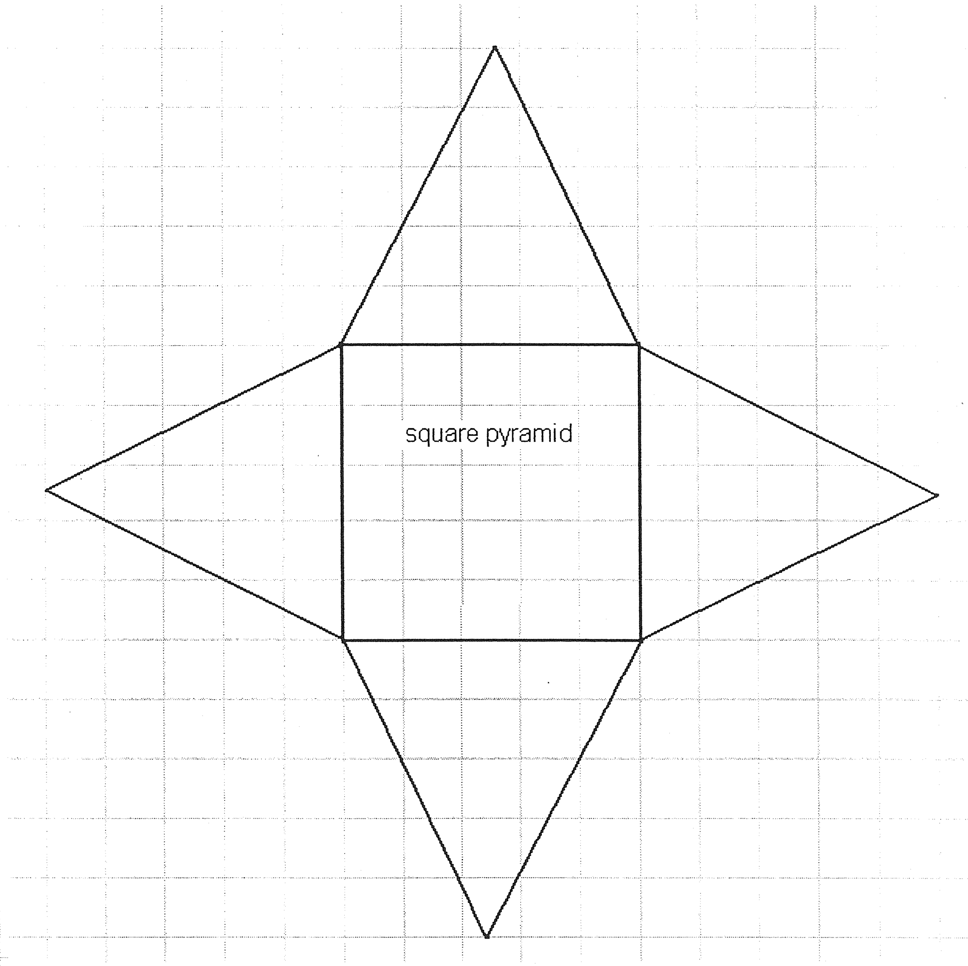
Justify your answer. How do you know?

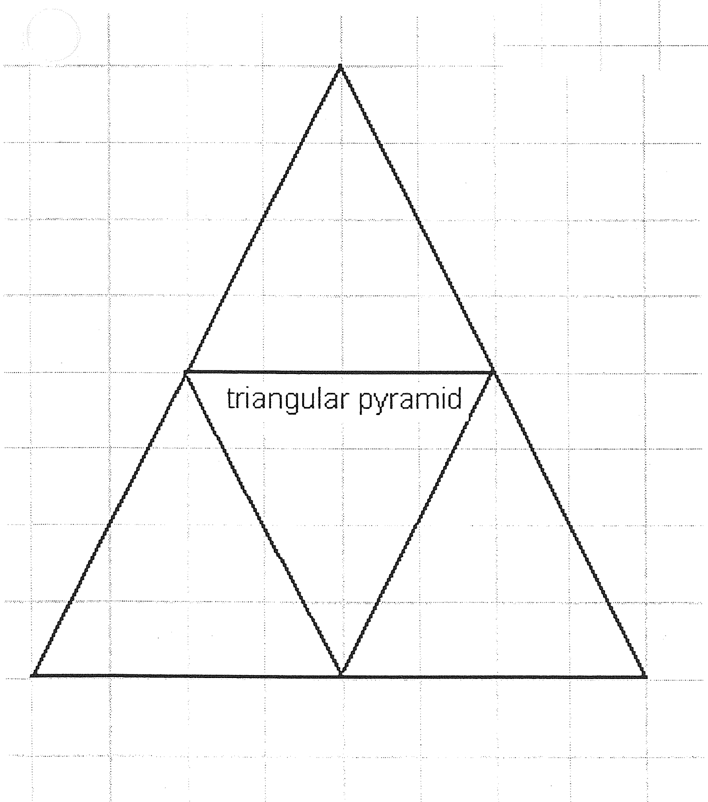
Name: ______________________________________ Date: ______________________ Period: _______

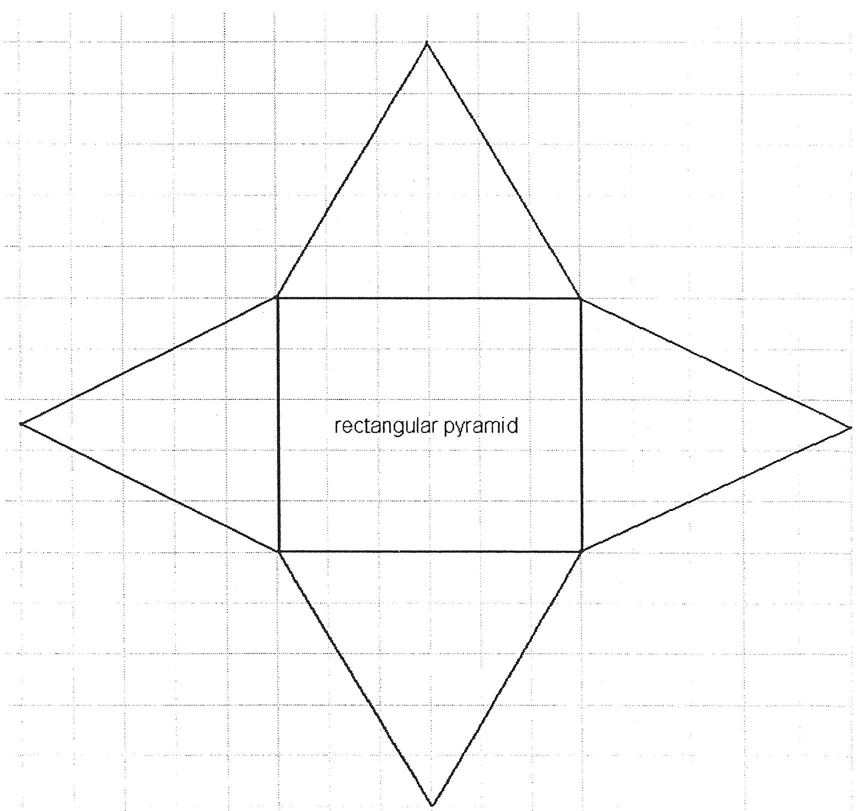
6th Grade Filling and Wrapping

Surface Area of Pyramids

Labsheet







Name ______________________

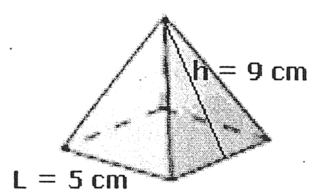
Surface Area of Triangular and Rectangular Pyramids

Period _____ Date ______________

Surface area of a Pyramid

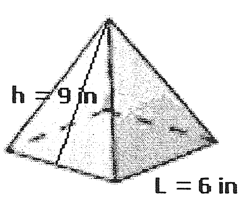
SA = sum of the area of all the faces

- A square pyramid has a base that is a ____________ and _____ triangle faces. There are _____ total faces.
- A ______________ pyramid has a base that is a triangle and _____ other triangle faces. That gives a total of _____ faces. All of the triangular pyramids are made of 4 congruent equilateral triangles.
- To find the Surface Area of any Pyramid, find the area of each face and add all the face areas together.
- Surface area is measured in square units.



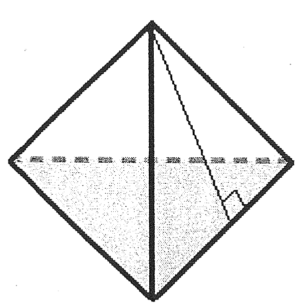
The base of the pyramid is a square. We call it a _____________pyramid.

Find the surface area of the pyramid.



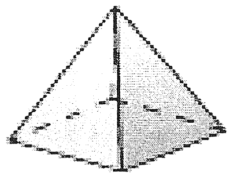
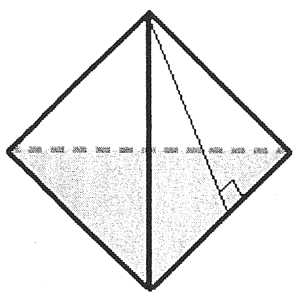
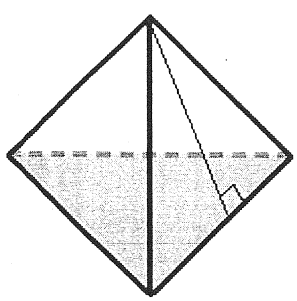
The base of this pyramid is a square. The figure is a ___________ pyramid.

Find the surface area of the pyramid.

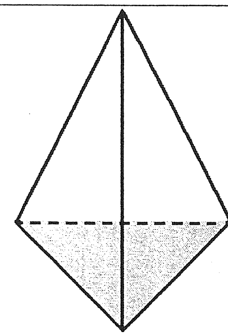
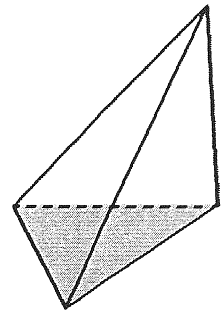
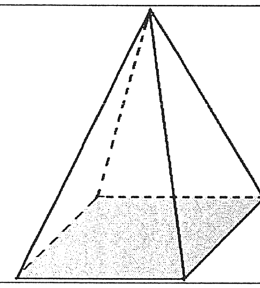
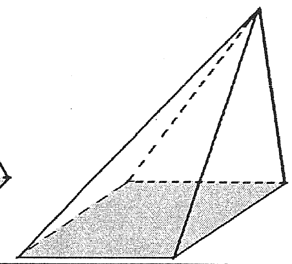
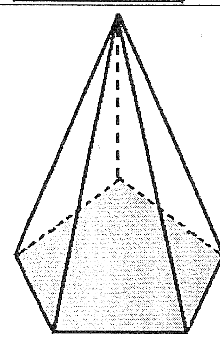
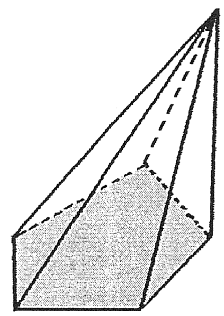
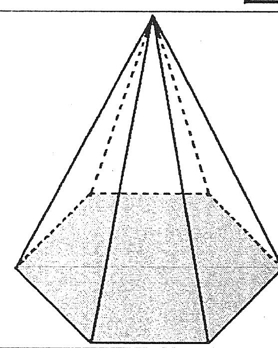
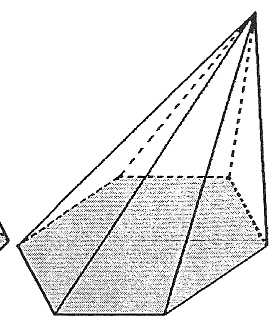
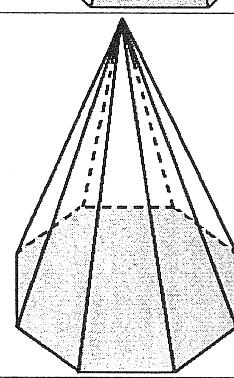
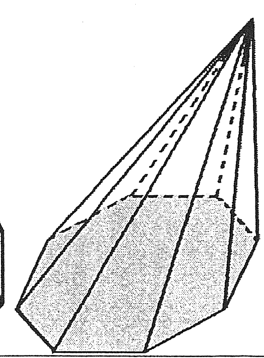


The figure is a ______________ pyramid. Each face has a base of 6 cm and a height of 5 cm

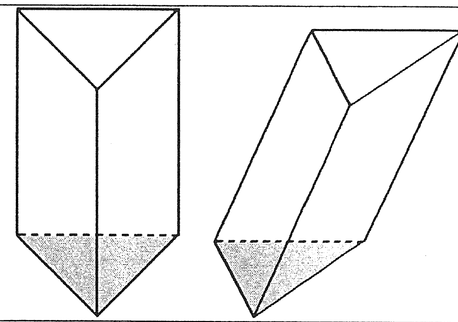
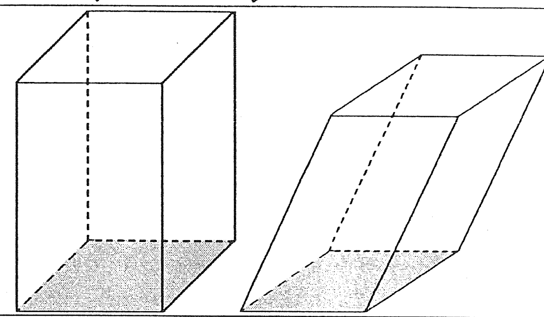
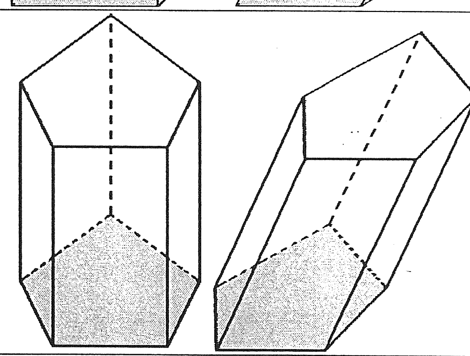
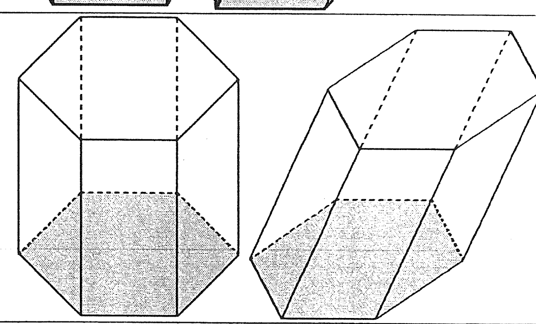
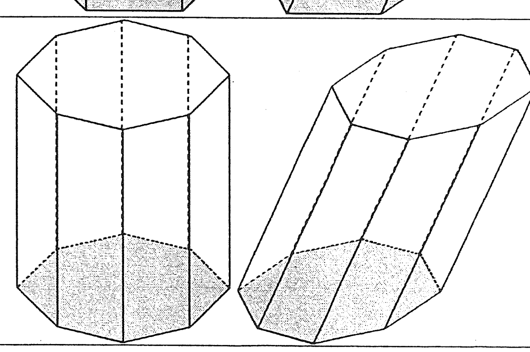
Find the surface area of the pyramid.

	The base of the pyramid is a square. Each side is 4 inches long. The height of the triangle faces are 6 in. The figure is a ______________ pyramid. Find the surface area of the pyramid.
	The faces of this triangular pyramid are all congruent. Each triangle has a base of 5 cm and a height of 4.25 cm. Find the surface area of the pyramid.
	The figure is a ______________ pyramid. Each of the faces are congruent. Each face has a base of 4 in. and a height of 3.5 in. Find the surface area of the pyramid.

Pyramids

Name	Shape of Base	Number of lateral Faces	Total number of faces	Examples
Triangular pyramid	triangle	3	4	 
Rectangular pyramid	rectangle	4	5	 
Pentagonal pyramid	pentagon	5	6	 
Hexagonal pyramid	hexagon	6	7	 
Octagonal pyramid	octagon	8	9	 

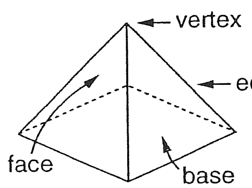
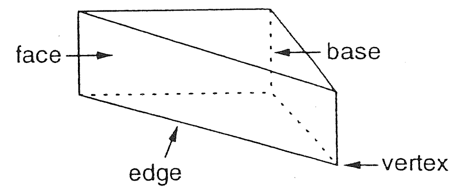
Prisms

Name	Shape of bases	Number of lateral faces	Total number of faces	Examples
Triangular prism	triangle	3	5	
Rectangular prism	rectangle	4	6	
Pentagonal prism	pentagon	5	7	
Hexagonal prism	hexagon	6	8	
Octagonal prism	octagon	8	10	

Name ______________________________

Polyhedron Parts

All polyhedrons have bases, faces, edges, and vertices. Two faces on a polyhedron meet at an **edge.** Three or more edges meet at a **vertex.**

Pyramid	Prism
 A pyramid has 1 base.	 A prism has 2 bases.

Complete the table. Write the number of bases. Find the missing numbers by using the formula **F + V = E + 2,** where F is the number of faces, V is the number of vertices, and E is the number of edges.

	Polyhedron name	Number of bases	Number of faces	Number of vertices	Number of edges
A.	triangular prism		5	6	
B.	triangular pyramid			4	6
C.	rectangular prism		6		12
D.	rectangular pyramid			5	8
E.	pentagonal prism		7	10	
F.	pentagonal pyramid		6		10
G.	hexagonal prism			12	18
H.	hexagonal pyramid		7	7	
I.	octagonal prism		10		24
J.	octagonal pyramid		9	9	

How Likely Is It?

Investigation / Lesson / Assessments	# of days	Resource Locations	Follow Up?	6-8 Performance Expectations
**Be sure to use the vocab of 6.3.G throughout the unit. Complimentary probability and also focus on probabilities written as fractions, percents, and decimals.*				
Prob. 1.1 Flipping for Breakfast pg. 5	1		must do	**6.3.F Determine the experimental probability of a simple event using data collected in an experiment.**
Prob. 1.2 Analyzing Events pg. 7	1		optional	
Inv. 1 Math Reflections pg. 13	1			
Prob. 2.2 Pondering Possible and Probable pg. 16	1		should do	**6.3.G Determine the theoretical probability of an event and its complement and represent the probability as a fraction or decimal from 0 to 1 or as a percent from 0 to 100.**
Inv. 2 Math Reflections pg. 21 (only do #3 and #4)	1			
Prob. 3.1 Bargaining for a Better Bedtime pg. 22	1		must do	
Check-up #1	1			
Prob. 4.1 Predicting to Win pg. 29	1		must do	
Prob. 4.2 Drawing More Blocks pg. 32	1		should do	Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Prob. 4.3 Winning the Bonus Prize pg. 33	1		should do	
Inv. 4 Math Reflection pg. 41 #1-3	1			
Review for Unit Assessment for How Likely Is It?	1			
Unit Assessment for How Likely Is It?	1			
Total Instructional Days for Probability:	**13 days**			

All page numbers given match the student texts.

Contents in How Likely Is It?

- There isn't any supplemental material for this unit